Suitability of two parameter distributions for low flows in Ireland and Wales

by

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Outline

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• Probability Plots
  – EV1
  – Lognormal
  – Descending order
• PPCC
• L-Moment Ration Plots
• Conclusions
Abstract

• The distribution of annual minimum one day low flows has been examined for a small selection of gauging stations in Ireland and Wales. Generally the observed samples suggest positively skewed distributions. While the lognormal distribution is a reasonable descriptor the EV1 distribution is also a good candidate. This may be contrary to expectations as the EV1 is known to be a distribution of maxima, not minima.
Source Data

• 28 series of annual minimum 1 day flows from Ireland, with
  – record lengths varying from 28 to 47 years
  – catchment areas varying from 106 to 1943 km²

• 12 series of annual minimum 1 day flows from Wales, with
  – record lengths varying from 32 to 45 years
  – catchment areas varying from 43 to 893 km²
Seasonal Distribution of Low Flows

• While low flows are expected to occur in August or September the annual minimum flow in Ireland can occur over a surprisingly wide range of times throughout the year – even in winter. Catchments with impermeable soils are most likely to show such a wide range of season.

• This is because any period of 3 to 5 weeks of below average rain can produce low flows. Such periods can occur in Spring as well as in Summer or Autumn.
Data Skewness

- Skewness can provide a clue as to the requires form of distribution.
- Annual minimum 1 day low flow series have positive skewness, with average Hazen corrected values of
  - 1.30 for 28 Irish stations
  - 0.79 for 12 Welsh stations
- It should be noted of course that sample skewness has large standard error
2 Parameter Distributions

• The EV1 and LN2 distributions have been tested initially for their suitability.
• EV1 has skewness of 1.14, somewhat similar to the average value among Irish data and somewhat larger than the average among Welsh data
• LN2 has a variable skewness and can usually describe fairly well data that are mildly positively skewed
Basis of Assessment

• Suitability is assessed on the basis of
  – Visual assessment on Probability Plots
  – Probability Plot Correlation Coefficient
  – L-Kurtosis versus L-Skewness plots
Probability Plots

• Data from 23 out of 28 Irish stations showed remarkably good straight line patterns on EV1 plots

• Data from 5 of the stations would be best described by a curve rather than a straight line, i.e. not EV1. Plots of censored data show improvement

• All of the 12 Welsh stations showed good straight line patterns on EV1 plots
Irish data – Good EV1 fit

Annual Minimum Flows Barrow at Levistown 1969-2002 on EV1 y(i) base
Same data – Lognormal plot

Log 10 Annual Minimum Flows Barrow at Levitstown 1969-2002 on $N(0,1)$
y base

Log 10 Discharge

$N(0,1)y$

-4.0 -3.0 -2.0 -1.0 0.0 1.0 2.0 3.0 4.0
Same data – descending order
No Improvement!

Annual Minimum Flows Barrow at Levistown 1969-2002 on EV1 y(i) base

Discharge (cumec)

EV1 y
Irish data – bad EV1 fit

Annual Minimum Flows Finn at Anlore 1957-2000

Discharge (cumeq)

EV1 y
Same data – Lognormal plot

Log 10 Annual Minimum Flows Finn at Anlore N(0,1) y base
Same data - descending

Finn at Alore Annual Minimum 1957-2000 on EV1 y(i) base Descending order

R^2 = 0.5884
Welsh data – EV1 plot

60006 Glangwili 1969 to 2004

Ann Min Flow, cumec

EV1 y
Welsh data – lognormal plot

$Z = \log_{10}(\text{AnnMin Flow})$
Probability Plot Correlation Coefficients (PPCCs)

- PPCCs rejected the EV1 hypothesis in only 2 out of 28 Irish stations despite there being strong visual evidence that EV1 is unsuitable in 5 cases.
- PPCCs did not reject the EV1 hypothesis in any of the 12 Welsh stations.
- PPCC does not seem to have good discriminatory power.
L-Kurtosis versus L-Skewness

- The data from the 28 Irish stations show considerable scatter.
- This is not solely due to the poor EV1 fit in 5 cases.
- While L-moments are quite robust some data series have some very low outliers, i.e. almost zero flow, and these have quite an effect.
- Overall data quality is therefore an issue.
Irish Data
L-Kurtosis versus L-Skewness

• The data of the 12 Welsh stations show more scatter than would be expected from a 2-parameter parent.

• However the mean of the points plots very close to the single point which represents the EV1 distribution
Welsh Data

![Graph showing Welsh data with various distributions such as Normal, EV1, Exp, GLO, GEV, LN3, and P3. The graph plots L-Skewness against L-Kurtosis.]
Ascending or Descending Series

- Probability plots of low flows have often traditionally been shown with data ranked from largest to smallest.
- This was probably done so as to have the smaller values displayed where the return period scale is most elongated if an EV1 base is used.
- However this implies that the data have negative skewness, contrary to what has been found, and not surprisingly the probability plots do not display straight line patterns.
Ascending or Descending Series

Annual Minimum Flows Barrow at Levistown 1969-2002 on EV1 y(i) base

Discharge (cumec) vs. EV1 y
Conclusions

• Skewness values support EV1 but do not rule out LN2
• Probability plots favour EV1 in all but 5 out of 40 cases
• However L Moment ratio diagrams do not support a single choice of distribution especially in the Irish case.
• Perhaps a single distribution may not be appropriate!
The End

Thank you.
Welsh Data

![Welsh Data](image_url)