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## APPLICATION OF AN ANALYTICAL MODEL TO OBTAIN DAILY FLOW DURATION CURVES IN PORTUGAL

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### Abstract

This work assesses the performance of an analytical model framework to generate daily flow duration curves, FDCs, based on climatic characteristics of the catchments and on their recession coefficients. According to the analytical model framework, precipitation is considered to be a stochastic process, modeled as a marked Poisson process, and recession is considered to be deterministic, with parameters that can be computed based on different models.

The analytical model framework was tested for two case studies located in mainland Portugal. For that purpose, three time intervals were analyzed (the wet and the dry semesters and the full hydrological year) and two developments of the model were tested: one considering a linear recession model and the other adopting a nonlinear recession model. Those developments were combined with recession coefficients obtained from two different approaches (forward and inverse estimation).

The performance of the analytical framework when considering forward parameter estimation is poor for both linear and nonlinear models (similar values the performance indicator). The inverse estimation approach shows exceptional good results, especially for the nonlinear model, clearly suggesting that the model has the ability to describe FDCs provided that the “right” values of the parameters are accounted. This circumstance indicates the need to develop additional research focused on the parameter estimation based on the available data.

**Keywords:** Analytical flow duration curves, stochastic process, linear and non-linear recession, hydrological modelling, daily discharges.

**Tema:** Hidrologia, hidráulica fluvial e obras hidráulicas.

## 1. INTRODUCTION

A flow duration curve (FDC) is a common representation of the availability and variability of daily discharges in a given river section. It is useful for many engineering applications, such as the design of small hydropower plants or water supply systems, studies about stream ecology alterations and sediment transport and water quality and allocation (Vogel and Fennessey, 1995). FDCs can be obtained empirically, by assigning empirical probabilities to observed ranked daily discharges (Vogel and Fennessey, 1994), or by using models that generate the curves based on hydrological variables other than discharges (Castellarin et al., 2013). An important category of the models able to predict FDCs are the process-based ones that combine climate controls and catchment characteristics to estimate FDCs. They can be based on long-term simulations of daily discharges or on the parametrization of the curves calculated from few key hydrologic controls, such as the model framework that is the object of this study.

Botter et al. (2007c) assumed that precipitation can be modeled as a stochastic process (i.e. a marked Poisson process) and be combined with a linear recession model to obtain FDCs that follow a gamma distribution. This approach only requires a few parameters: the mean depth of daily precipitation, the frequency of precipitation events that produce discharge, the area and the mean residence time (the inverse of the linear recession coefficient) of the catchment. This original framework has already been extended to embrace other hydrological conditions, such as nonlinear recession models (Botter et al., 2009), snow accumulation during winter (Schaeffli et al., 2013), urbanized catchments (Mejía et al., 2014) and dry climates (Müller et al., 2014). The original model framework and its extensions have already been applied successfully to catchments in the US (Basso et al., 2015, Botter et al., 2013, Ceola et al., 2010, Müller et al., 2014), Italy (Botter et al., 2007b), Switzerland (Basso et al., 2015, Doulatyari et al., 2015, Schaeffli et al., 2013) and Nepal (Müller et al., 2014), but it was never tested for Portugal.

Process based frameworks, such as the one under study, have the advantages of providing an explicit link between the FDC shape, rainfall characteristics and catchment recession characteristics and of being applicable to periods characterized by different meteorological conditions. Besides, this framework is also very simple and does not require much data or computational capacity.

The objective of the present work is to test the performance of an analytical model framework to obtain seasonal and annual FDCs in two Portuguese catchments: Vinhais Quinta da Ranca and Vale Giestoso river gauge stations. The applications considered as time intervals to which the FDCs refer the wet and dry semesters and the full hydrologic year. The recession parameters were calculated based on the assumption of linear and nonlinear recession models. The calculation of parameters followed two different approaches: a forward estimation, which consists in computing the parameters directly from the data, and the inverse estimation, which applies a calibration procedure thus resulting in optimized parameters.

The paper is organized as follows: Section 2 provides a description of the analytical model framework, together with the methods adopted, followed by the presentation of the case studies (Section 3). The results obtained are presented and discussed in Section 4 and conclusions are summarized in Section 5.

## 2. METHODS

This section begins with a short overview of the modeling framework applied to Portuguese catchments, followed by a description of the methods used to estimate the model parameters and to assess the model performance.

### 2.1 Model framework

The analytical model framework for probabilistic characterization of rainfall-driven daily discharges developed by Botter et al. (2007c) is based on a previous model proposed by Rodriguez-Iturbe et al. (1999). This original model represents the dynamics of soil moisture at a point as a result of a deterministic state-dependent loss function, combined with stochastic increments triggered by rainfall events. Accordingly, Botter et al. (2007c) proposed to describe the dynamics of daily stream flow supposing that some precipitation events act as a stochastic forcing for discharge production and that water is released from the soil producing discharge according to a deterministic recession.

It is assumed hereby that discharges ( $Q$ ) are the result of a sequence of subsurface inputs triggered by precipitation events that deliver enough water to fill the water deficit in the soil and to raise its level of moisture above its retention capacity. The excess of water becomes discharge and is removed from the soil as subsurface run-off. Originally, the subsurface storage was assumed to behave like a linear reservoir with a constant  $k_l$ . The overall rainfall forcing can be modeled as a marked Poisson process with frequency  $\lambda_P$  and exponentially distributed rainfall depths with average  $\alpha$ . But not all the rainfall events produce discharge because of the losses due to evapotranspiration and retention in the soil. Accordingly,  $\lambda_P$  is reduced to  $\lambda$ : the frequency of discharge-producing events, i.e., of events that raise the soil moisture above its retention capacity. In rainfall-driven environments, the  $\lambda$  reduced frequency can be understood as the frequency of rainfall events that are unusable by plants; is influenced by the soil storage capacity and soil drying time (Botter et al., 2007b). From those assumptions, it is possible to obtain the following probabilistic distribution for daily discharges that has the shape of a gamma distribution:

$$p(Q, t \rightarrow \infty) = \frac{1}{\Gamma\left(\frac{\lambda}{k_l}\right)} \frac{1}{Q} \left(\frac{Q}{\alpha k_l A}\right)^{\frac{\lambda}{k_l}} \exp\left(-\frac{Q}{\alpha k_l A}\right), \quad (1)$$

where  $A$  is the catchment area and  $\Gamma$ , the gamma function. Botter et al. (2009) extended this framework to consider a nonlinear recession and obtained a new equation to describe daily discharges:

$$p(Q, t \rightarrow \infty) = C \left\{ \frac{1}{Q^a} \exp \left[ -\frac{Q^{2-a}}{\alpha k_n (2-a)} + \frac{Q^{1-a} \lambda}{k_n (1-a)} \right] \right\}, \quad (2)$$

where  $a$  and  $k_n$  are the nonlinear recession coefficients (for a recession represented by  $dQ/dt = k_n Q^a$ ) and  $C$  is a normalizing constant (Botter et al., 2009).

## 2.2 Time intervals

The model is suitable for steady state conditions, at the annual or seasonal scales, depending on the temporal variability of the model parameters (Botter et al., 2007a). The most common periods of application are the four meteorological seasons (Botter et al., 2007b, 2013, 2008, Ceola et al., 2010), but can be different from those.

In Portugal, the hydrological regimes can be classified as pluvial, which means that the trigger to discharge production is rainfall. It is also known that the precipitation has a marked seasonal behavior, with a wet semester and a dry semester, which affects the values of the parameters of the model. Taking into account that seasonality, the model framework was applied to three different time intervals: the full hydrologic year (01-Oct to 31-Sep) and the two previous semesters (from 01-Oct to 31-Mar and from 01-Apr to 31-Sep, respectively).

## 2.3 Parameter estimation

The model parameters are related either to the stochastic inputs or to the deterministic recession. For each time interval, all the parameters were firstly calculated in a forward mode, i.e., directly from data, without calibration. Additionally, the recession parameters were also calibrated to optimize the results. The calibration was performed by fixing the stochastic inputs' parameters and optimizing the recession parameters using maximum likelihood estimates.

Stochastic inputs' parameters are the mean depth of precipitation ( $\alpha$ ) and the frequency of the events that produce discharge ( $\lambda$ ).  $\alpha$  can be obtained as the mean of the positive daily effective precipitation. Effective precipitation is obtained simply by subtracting a maximum of  $1mm$  from each daily precipitation record.  $\lambda$  is obtained from a combination of the remaining daily precipitation data and the equivalent daily discharges from the relation  $\bar{Q} = \lambda \alpha$ , where  $\bar{Q}$  is the long term average of the observed daily discharges  $Q$  in the time interval being considered. The estimation based on this method has been shown by Ceola et al. (2010) to provide the best results, and it is used by the majority of studies since then (Ceola et al., 2010, Botter et al., 2013, Basso et al., 2015). Deterministic recession parameters are obtained by means of recession analysis, which comprehends two steps: recession extraction and parameter estimation. Recession extraction refers to the selection of discharge data in periods when the

only source of stream flow is the water stored in the soil. Such data will be used in the parameter estimation step.

Hereby, the method applied for recession extraction followed the work of Dralle et al. (2017), who suggest that recessions should be selected based on an upward concavity requirement in the hydrographs, with a minimum length of four days. Those authors also studied the influence of a peak selectivity criteria, concluding that it does not interfere significantly in the results, so we have chosen a simple method, selecting only recessions that begin with a discharge higher than the annual or seasonal long term mean discharge, depending on the time interval under analysis. This type of criteria has been adopted previously by Biswal and Marani (2010), Mutzner et al. (2013).

Parameter estimation utilized the method proposed by Brutsaert and Nieber (1977) based on a linear regression of the  $\log(Q)$ - $\log(dQ/dt)$  of all selected data points. The values of  $Q$  are the means of each two consecutive days and are used to obtain the correspondent  $dQ/dt$ .

## 2.4 Performance evaluation

Taking into account that the FDC can be understood as a probabilistic distribution of daily discharges that can also be represented as cumulative distribution function (cdf), the Kolmogorov-Smirnov distance ( $c^{KS}$ ) was used to assess the performance of the model, as previously done by other authors (Ceola et al., 2010, Schaefli et al., 2013). The  $c^{KS}$  represents the maximum distance or probability gap between an analytical cdf derived from the model, and an empirical cdf.

## 3. CASES STUDIES

The two Portuguese catchments adopted as case studies are those at the river gauges stations of Vinhais Quinta da Ranca (QRA) and Vale Giestoso (VGI), located in the north of Portugal. The data was obtained from the SNIRH data basis and the adopted recording periods were from 01/10/1957 to 30/08/1989 (QRA) and from 01/10/1956 to 30/08/1985 (VGI). The weighted daily precipitation series were obtained by applying the Thiessen methods to the records at rain gauges identified in Table 1, together with some characteristics of the catchments.

**Table 1:** Characteristics of catchments and rain gauges

Catchment					Rain gauge		
Name (-)	Coordinates M/P(m)	Main basin (-)	River (-)	Area ( $km^2$ )	Name (-)	Coordinates M/P(m)	Weight (-)
Vinhais - Quinta da Ranca	294575/538511	Douro	Rio Tuela	455	Moimenta da Raia	295869/553749	0.490
					Montezinho	311835/552285	0.210
					Celas	300772/527894	0.070
					Vinhais	293547/541066	0.230
					Cervos	237648/529753	0.331
Vale Giestoso	235234/526829	Douro	Rio Beça	78	Barracão	235207/532671	0.354
					Firvidas	234153/535597	0.320

#### 4. RESULTS AND DISCUSSION

The values of the parameters and of the performance indicator  $c^{KS}$  for the two case studies are shown in the Table 2. When analyzing the table one should have in mind that the best values of the parameters are those obtained by inverse estimation. The cdfs derived from the FDCs for the linear and nonlinear models are presented in Figures 1 and 2, respectively.

**Table 2:** Parameters and  $c^{KS}$  values for each season and model. The lower indexes  $lf$ ,  $li$ ,  $nf$  and  $ni$  stand for linear forward, linear inverse, nonlinear forward and nonlinear inverse respectively.

Catchment and period	Common parameters			Linear model				Nonlinear model					
	$\alpha$ [1]	$\lambda_P$ [2]	$\lambda$ [3]	$k_{lf}$ [4]	$c_{lf}^{KS}$ [5]	$k_{li}$ [6]	$c_{li}^{KS}$ [7]	$k_{nf}$ [8]	$a_f$ [9]	$c_{nl}^{KS}$ [10]	$k_{ni}$ [11]	$a_i$ [12]	$c_{ni}^{KS}$ [13]
QRA													
Wet	9.857	0.435	0.339	0.074	0.370	0.427	0.076	0.061	1.395	0.342	0.378	1.330	0.024
Dry	6.751	0.243	0.165	0.062	0.348	0.235	0.097	0.064	1.117	0.350	0.346	1.483	0.078
Year	8.755	0.338	0.256	0.067	0.387	0.396	0.071	0.067	1.230	0.373	0.425	1.369	0.048
VGI													
Wet	10.074	0.454	0.319	0.066	0.405	0.408	0.080	0.057	1.295	0.393	0.332	1.484	0.029
Dry	6.596	0.234	0.160	0.059	0.283	0.157	0.095	0.055	0.904	0.293	0.197	1.497	0.046
Year	8.915	0.342	0.239	0.063	0.413	0.324	0.109	0.060	1.120	0.412	0.320	1.538	0.032

To begin with, we should stress the noticeable performance of the inverse estimation approach for the linear model (column [7] and blue dashed lines of Figure 1), but especially for the nonlinear model (column [13] and blue dashed lines of Figure 2), which clearly indicates that exceptional results can be obtained provided that the “right” values of the parameters are considered. The question still open is how to identify those values based on the available data.

For the linear model the forward estimation of the recession parameters - column [4] - provides values largely underestimated in relation to the parameters calculated by inverse estimation - column [6]- leading to poor performances (column [5] and red dotted curves of Figure 1). In the forward estimation, the best performance of the linear model for both catchments occurs in the dry semester – lowest values of column [5] – when the recession parameters are the smallest. In fact, only those performances have a quality close to the ones mentioned in previous works (Ceola et al., 2010). During the wet semester, the assumption of dominant sub-superficial flow may not be accurate enough which may justify the poor quality of the results.

For the nonlinear model the forward approach also underestimates the parameters – columns [8] and [11] and presents poor results (column [10] and red dotted curves of Figure 2). It is worth mentioning that the nonlinear exponent of the recession is close to one - column [9] -, leading to similar performances for the linear and the nonlinear models - columns [5] and [10].



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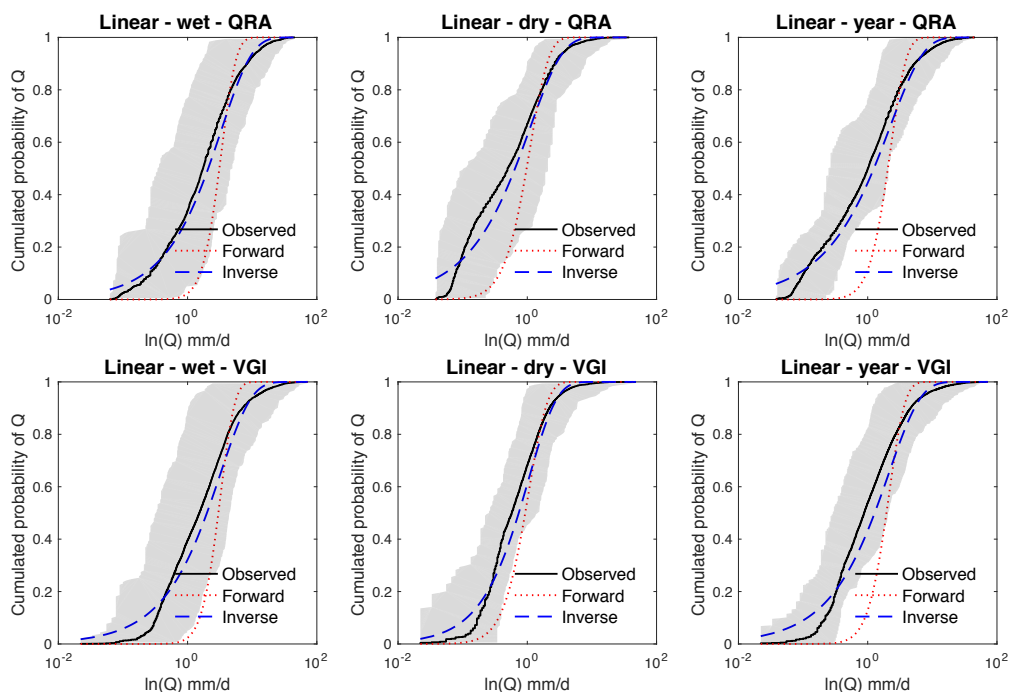


Figure 1: Cdfs for linear model.

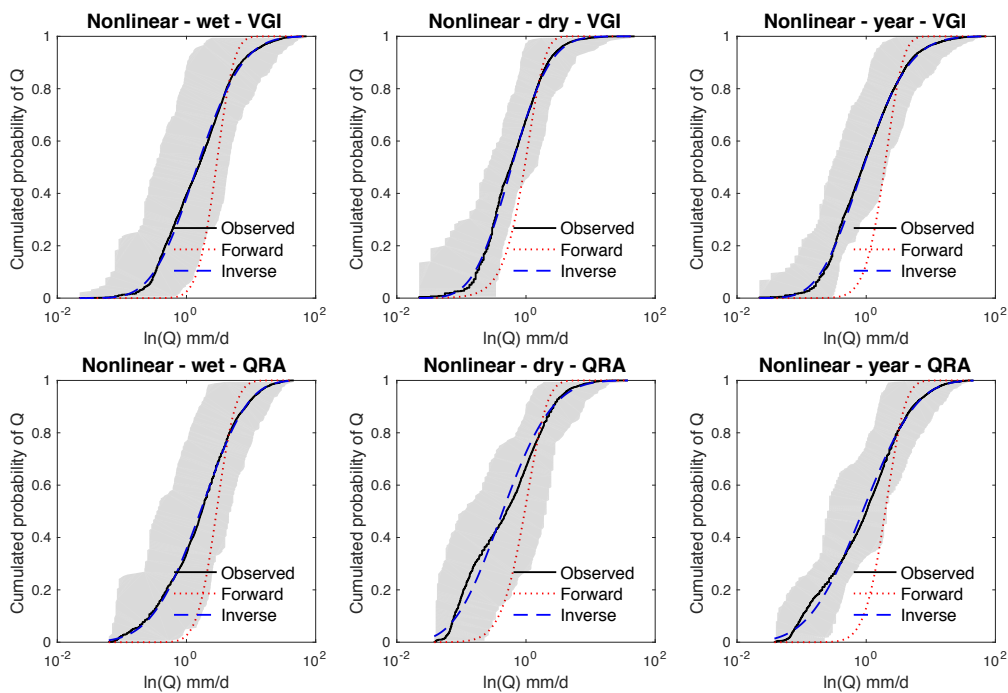


Figure 2: Cdfs for nonlinear model.



## 5. CONCLUSION

The following conclusions are worth emphasizing:

- The model performance is good for Portuguese catchments only adopting an inverse approach which restricts its applicability to catchments where some discharge data is available for parameter optimization.
- For the inverse estimation the performance of the nonlinear model is better than the one of the linear model, showing that this model has more potential to represent FDCs in Portugal.
- The performance of the forward estimation is always poor. Forward parameter estimation tends to largely underestimate the values of recession parameters in comparison to inverse estimation for both, linear and nonlinear models.
- In the forward estimation, the performances of linear and nonlinear are not significantly different, what can be explained by the values of the recession exponent  $a$  that is in general close to one.
- The good performance of the inverse estimation, especially when coupled with a nonlinear model, suggests that the model has the ability to describe flow duration curves provided that the correct values of the parameters are accounted for. That opens good perspectives for future research, namely focused on models for parameter estimation based on the available data.

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