1D Modelling of Floods in Rivers

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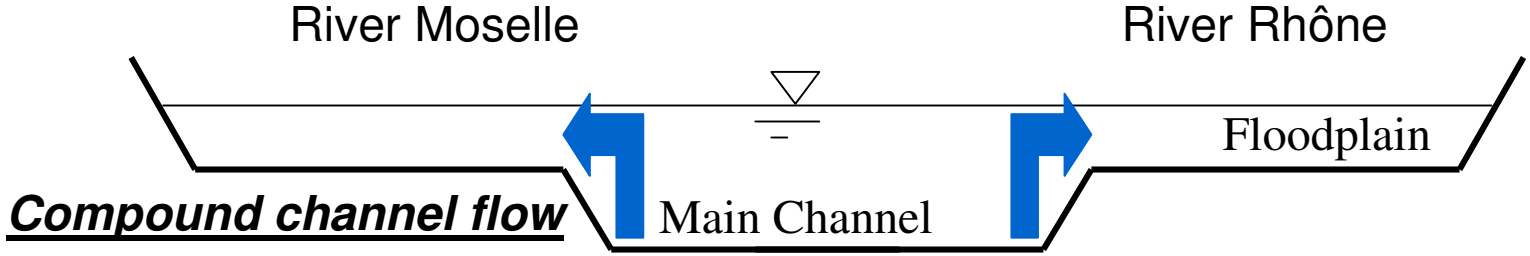

Compound channel flow

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Outline

- **1) Interest and scope of the topic.**
- **2) Physics of compound channel flows**
- **3) The coupling between water level and discharge distribution in 1D models** *(link between errors in water level and discharge in the floodplains).*
- **4) Validity of calibrating Manning roughness coefficient** (*examples of different geometries, water levels and numerical codes*)
- **5) The important issues in 1D modelling.**
- **6) An alternative approach based on three coupled 1D Saint-Venant equations.**
- **7) Conclusions and … open discussion**

1) Interest of the topic.

Flow rate at bank full level in the main channel :

Time = 15:43

Boscastle, UK, 2004

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1) Interest of the topic.

Time = 16:06 (+23 min)

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1) Interest of the topic.

16 h 50 (+ 1 h 07)

Vidéo

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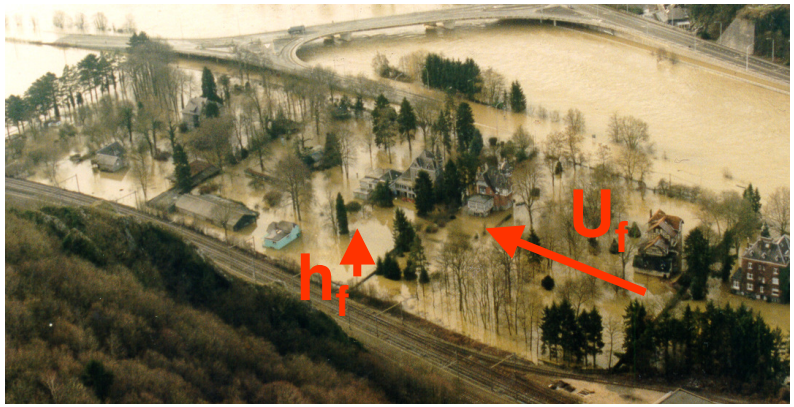
1) Interest of the topic.



2 challenging issues:

1) *Stage – Total discharge relationships (Q) ↔ (Z)*

- identify flooded areas (**Z**) from predicted discharges (**Q**)

- Calculation of the conveyance **Q** for a given $\mathbf{Z_{max.}}$

2) *Discharge Q_f and flow depth h_f in the floodplains*

- Urban areas : vulnerability ∝ $\mathbf{h_f} \times \mathbf{U_f}$

- Rural areas : mitigation schemes

⇒ Focusing on floodplain flow $\{Q_f, h_f\}$

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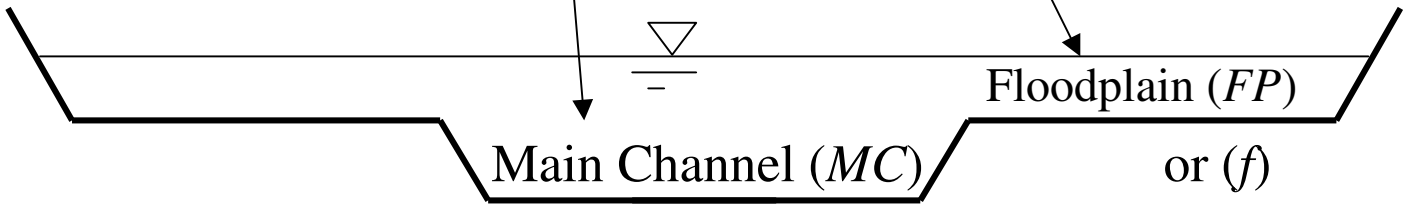
1) Scope of the topic.

Compound channel = Main channel + Flood Plains(s)

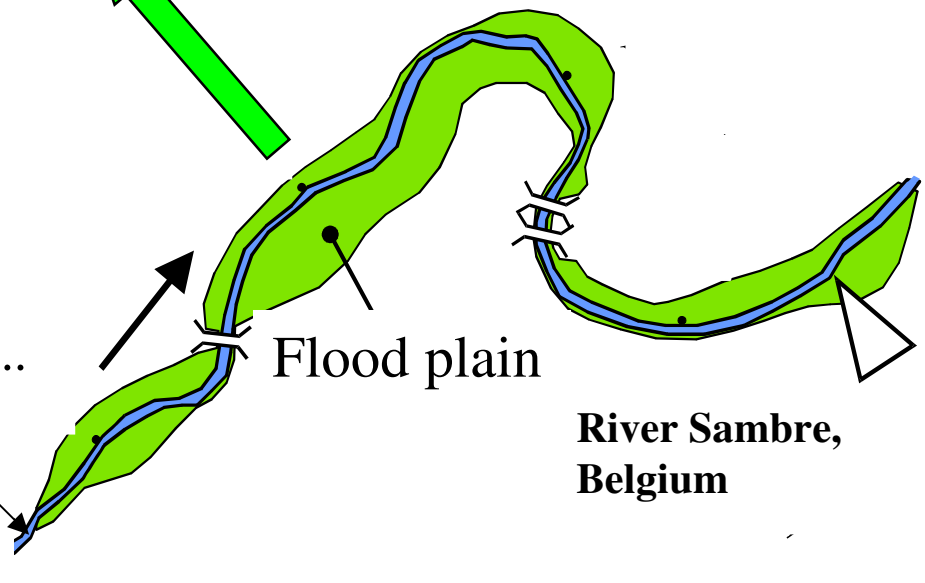


- **Variations in width of the flood plains**

√ Converging reaches

√ Diverging reaches

√ Presence of groynes, dikes..



River Sambre, Belgium

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1) Scope of the topic.

River Sambre, Belgium

- **Bed load transport** in the MC / + **sediments export** towards the FP

Scope of this presentation:

- Gradually varied flow in prismatic and non-prismatic geometries
- without movable sediment.
- without recirculation zone / flow separation (no groyne, dyke..)

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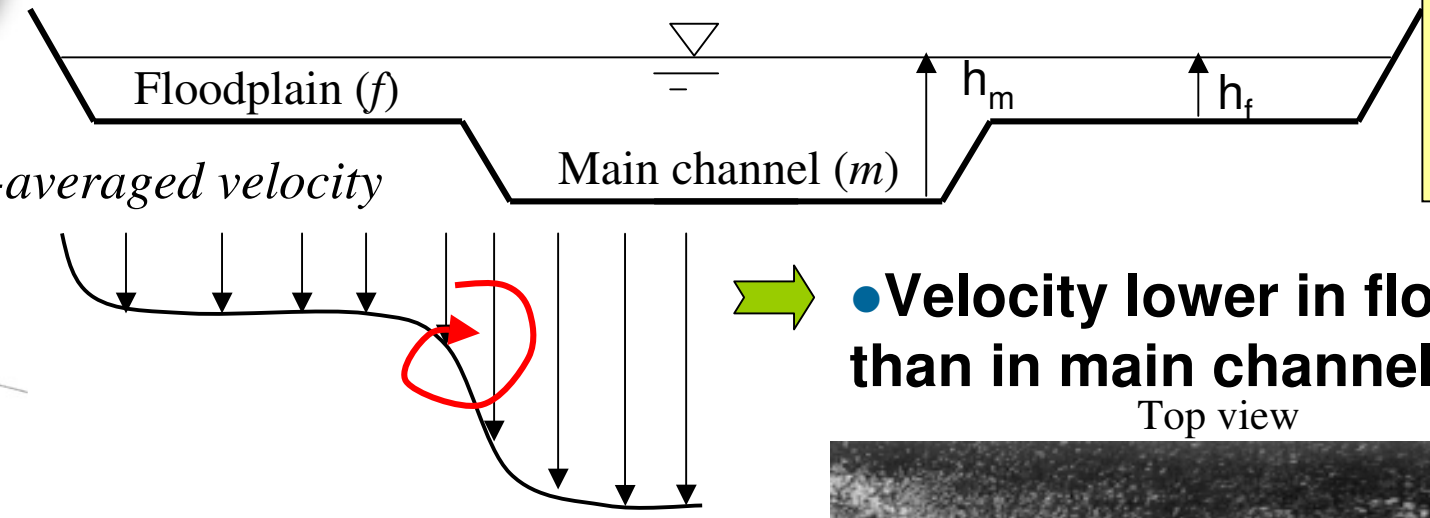
2) Physics of the compound channel flow

Uniform flow in Straight compound channel

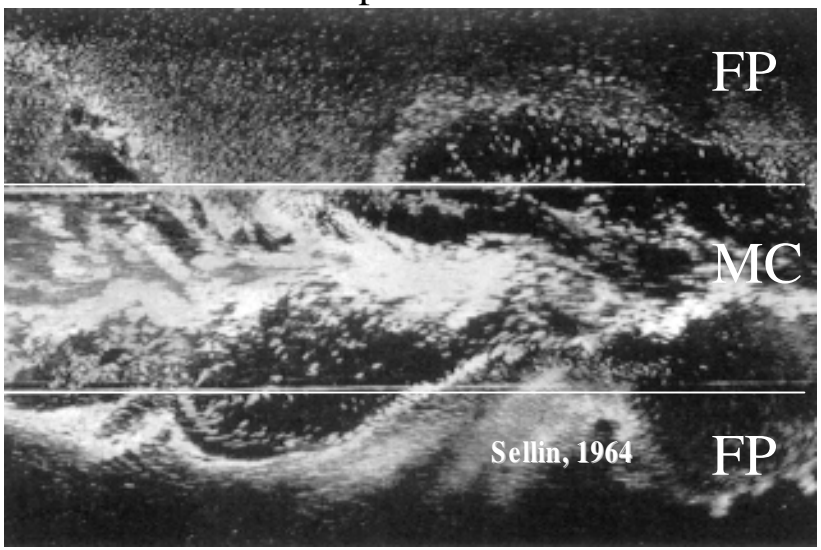


Relative depth

$h_r = h_f / h_m$

- **Velocity lower in floodplain than in main channel**

Top view



Turbulent exchange

√ Shear layer

√ Large vortices ↗ when h_r ↘

√ Momentum flux ⇒ **head loss**

⇒ Reducing Q_m and Q (for a fixed Z)

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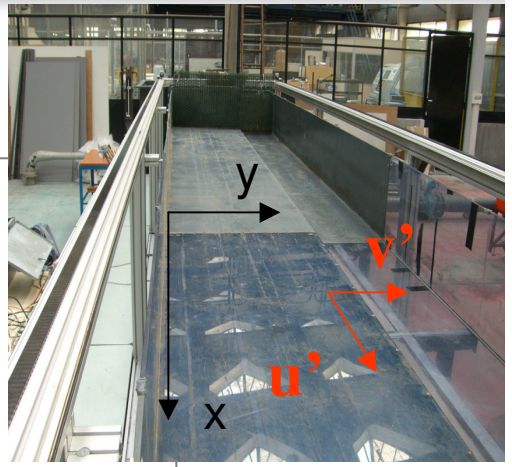
2) Uniform flow in straight geometry

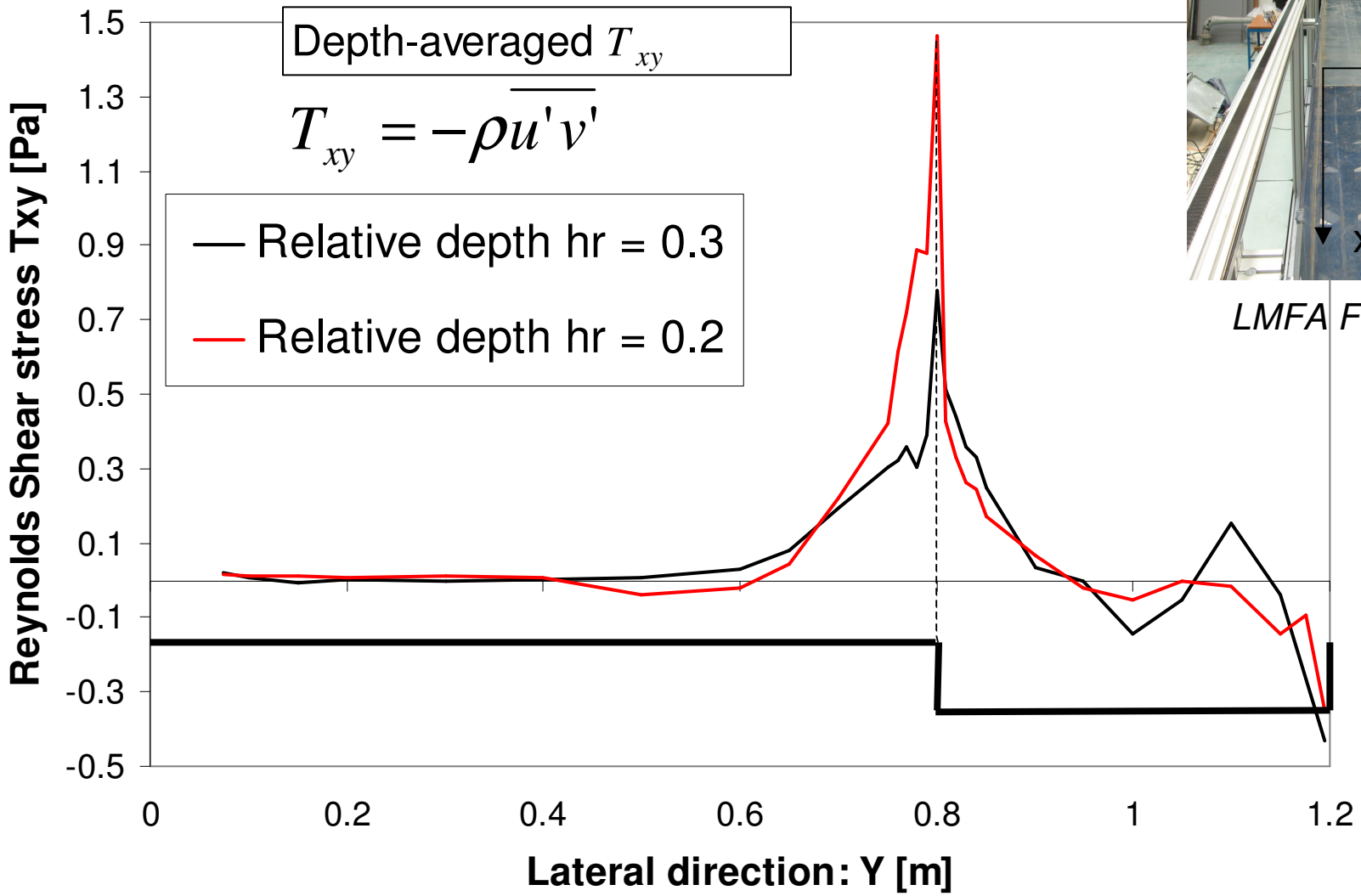
Head loss due to turbulent shear stress « T_{xy} »

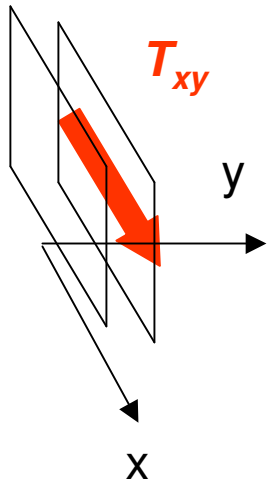


LMFA Flume, Lyon

Depth-averaged T_{xy}

$$T_{xy} = -\rho\overline{u'v'}$$





Peltier Y, PhD student at Cemagref, 2009

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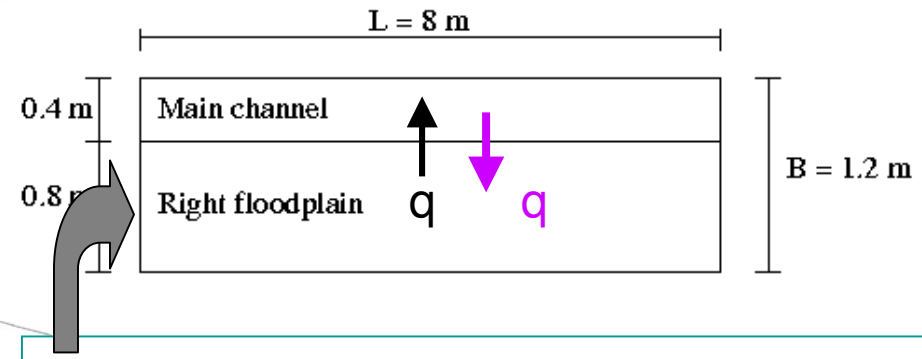
2) Non-uniform flow in straight geometry

Non-uniform flow: lateral mass discharge ***q***



LMFA flume

Straight compound geometry

Q_f (inlet) **exceeding** Q_f under uniform flow conditions

less than

Percentage increase / decrease in upstream floodplain flow
-19%
+19%
+38%
+53%

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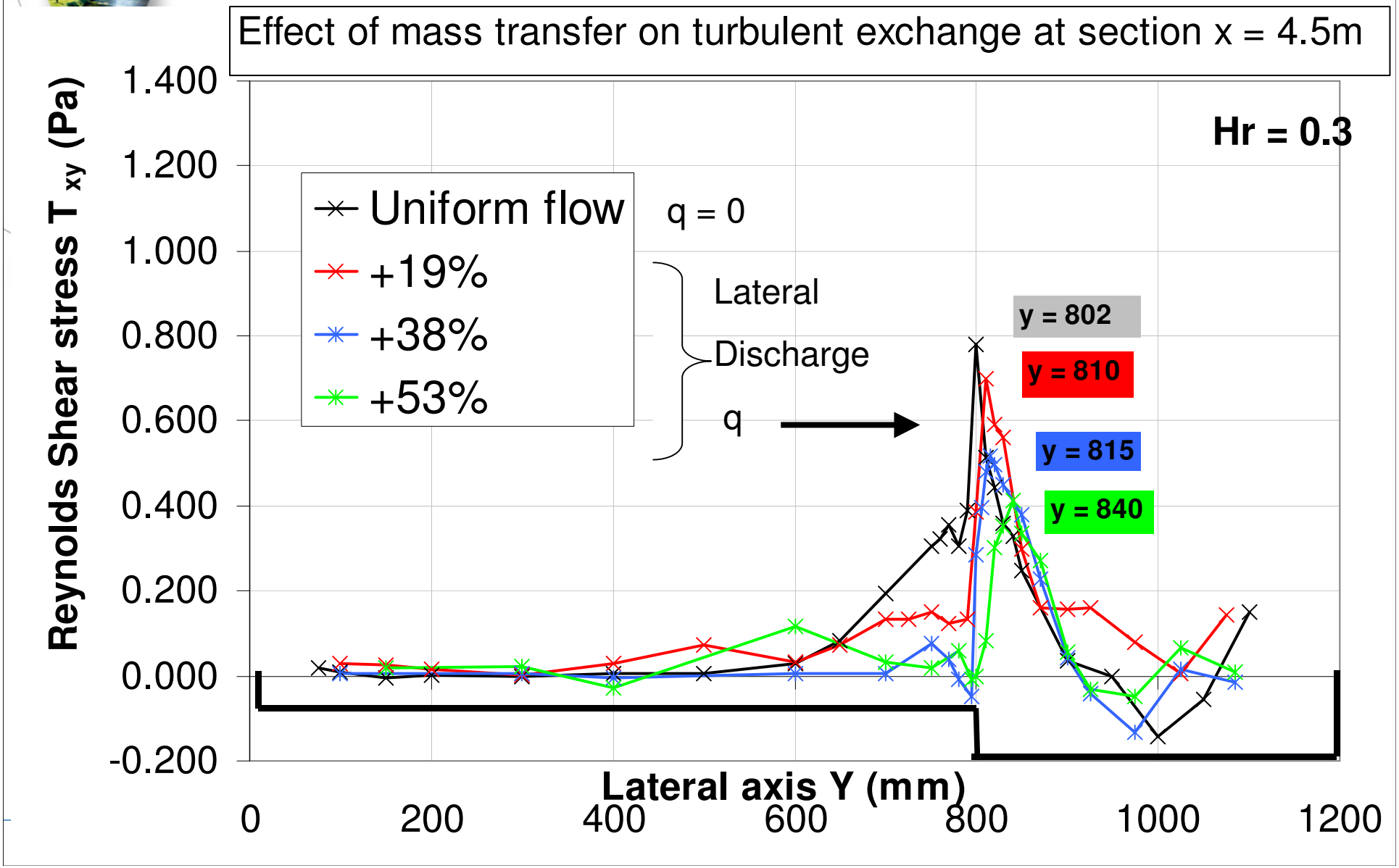
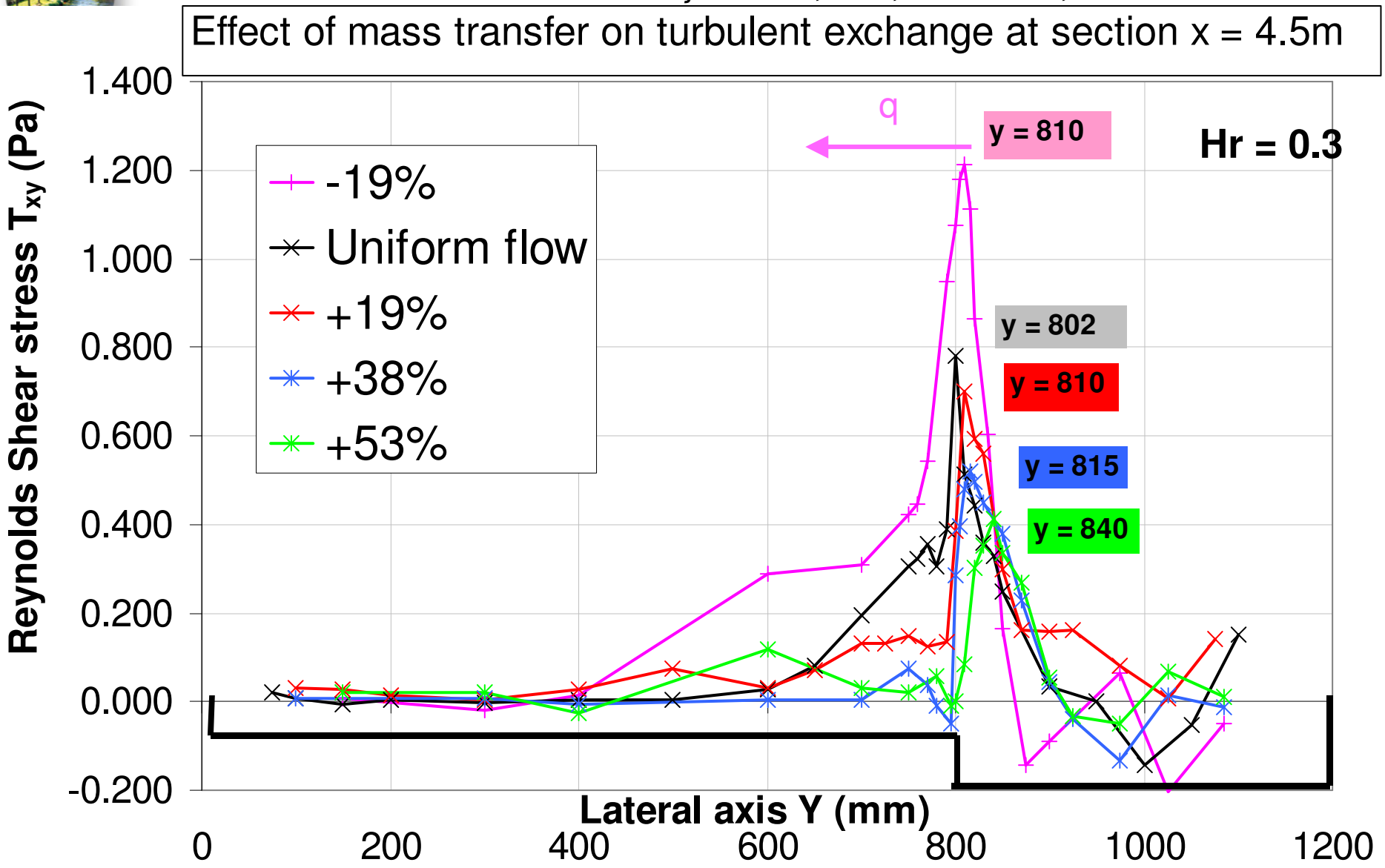
2) Non-uniform flow in straight geometry

Pessoa Project 2009, Leal, Fernandes, Proust



2) Non-uniform flow in straight geometry

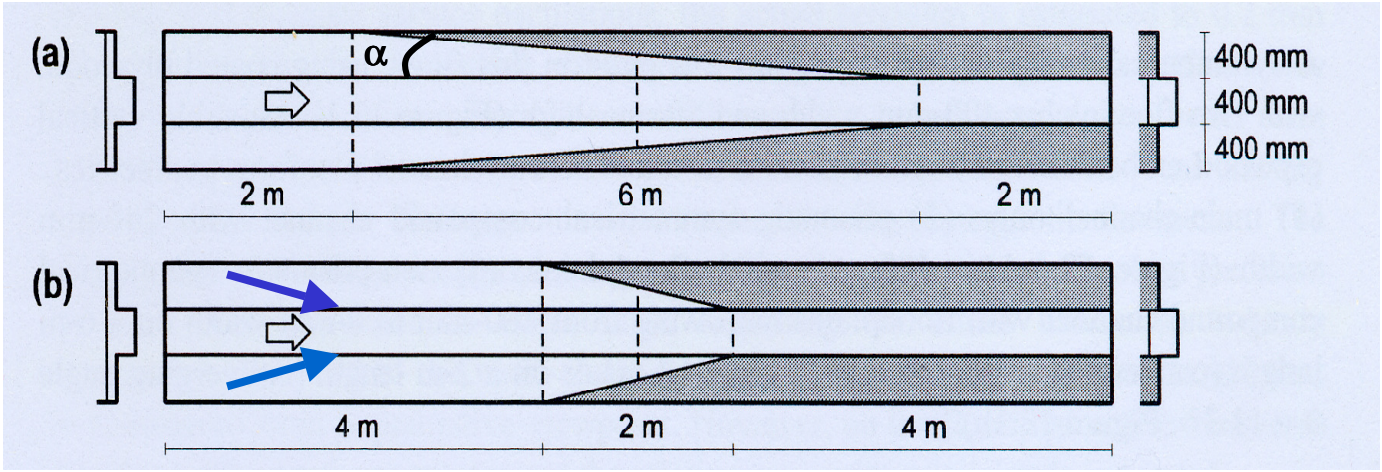
Pessoa Project 2009, Leal, Fernandes, Proust



2) Non-prismatic geometry

$\alpha = 3{,}8°$

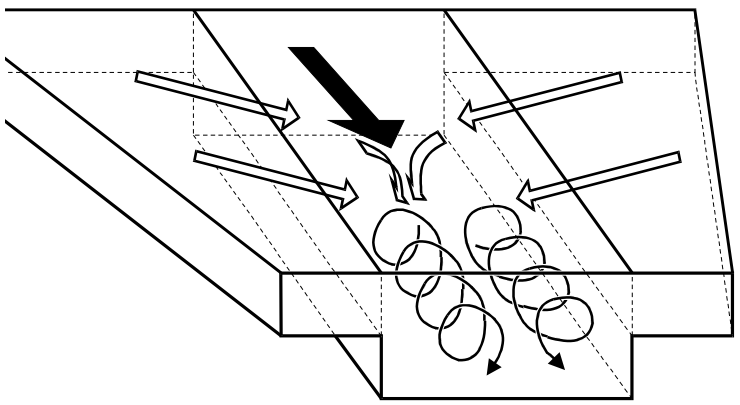
$\alpha = 11°$



√ **Strong** lateral mass exchange q

√ Strong variation in flow depth h(x)

Bousmar (2002)



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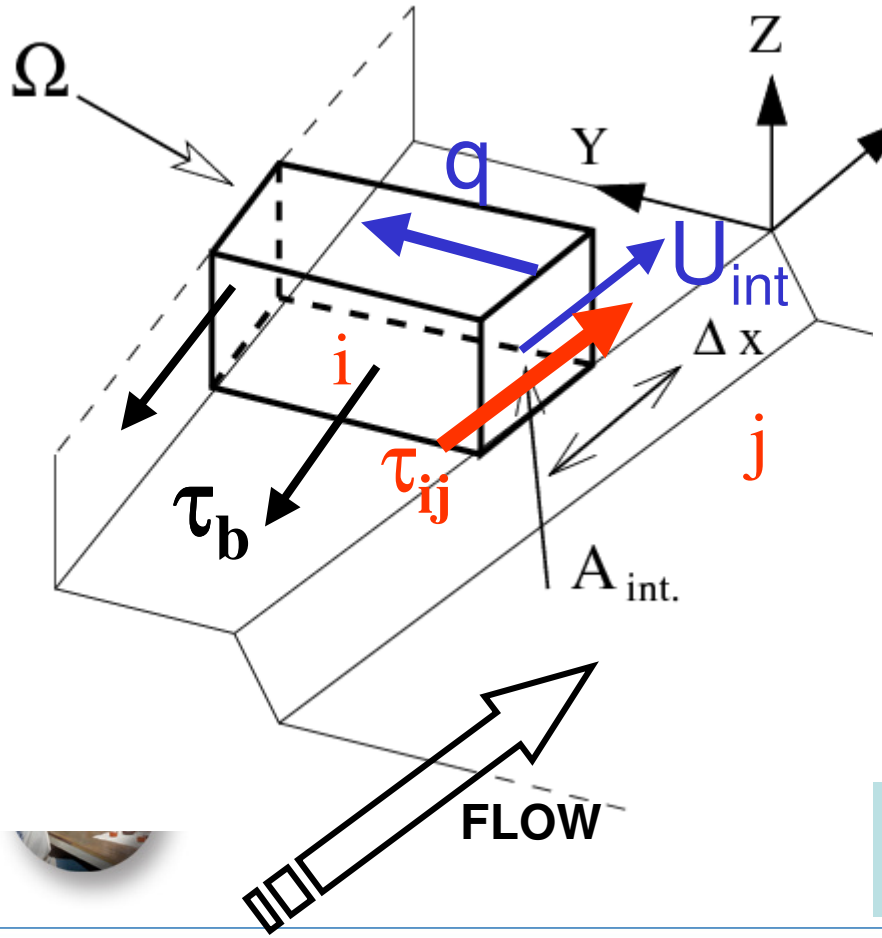
2) Overview of the physics ?

Momentum balance in one sub-volume Ω (MC, left or right FP)



√ **Bed shear stress τ_b**

√ **Turbulent shear stress τ_{ij} applied to A_{int}, the interface area between two sub-sections "i" and "j"**

√ **Momentum flux due to mass exchanges between i and j = " q U_{int}"**

U_{int}: depth-averaged value of the longitudinal velocity at the interface

2) Overview of the physics ?

Momentum conservation in Ω, of sub-section « i », area A_i :

$$S_{fi} = S_0 - \frac{dh_i}{dx} \pm \frac{\tau_{ij}.h_{\text{int}}}{\rho g A_i} - \frac{1}{gA_i}\frac{d}{dx}\left(A_i U_i^2\right) + \frac{\left(qU_{\text{int}}\right)}{gA_i} \qquad (1)$$

Mass conservation in Ω :

$$\frac{dA_i U_i}{dx} = q \qquad (2)$$

h_{int} = flow depth at the interface

Sf_i = subsection friction slope

S_o = bottom slope

Head loss in Ω :

Subsection Head

$$S_{Hi} = -\frac{dH_i}{dx} = S_{fi} \pm \frac{\tau_{ij} h_{\text{int}}}{\rho g A_i} + \frac{q\left(U_{\text{int}} - U_i\right)}{gA_i} = S_{fi} + S_i^t + S_i^m$$

2) Head loss in one sub-section

3 sources of head loss in each subsection "i"

√ **Bed friction slope (S_{fi})**

√ **Head loss/gain due to turbulent exchanges (S_i^t)**

√ **Head loss/gain due to mass exchanges (S_i^m)**

$$S_{Hi} = S_{fi} + S_i^{\,t} + S_i^{\,m}$$

1) **S_f, S^t and S^m** influence flow depth h_f, and discharge distribution $\{Q_m , Q_f\}$

2) Relative weights of **S_f, S^t and S^m** vary with:

- geometry (cross-sectional, in the horizontal plane), roughness…
- relative depth h_r, total discharge Q

Proust et al. (2009), Water Resource Research, in press.

Proust et al. (2009), Advances in Water resource, accepted for publication

3) 1D models on the total cross-section

1D Equations on the overall compound cross-section

- Saint-Venant $$\frac{1}{gA}\frac{d}{dx}\left(\beta A U^2\right)+\frac{dZ}{dx}+S_f=0$$

- Bernoulli $$-\frac{d}{dx}\left(Z+\alpha\frac{U^2}{2g}\right)=S_f+S_a$$

and

Additional head loss

$$\begin{cases} S_a = 0 \\ S_a = S^t \\ S_a = S^t + S^m \end{cases}$$

The correction coefficients of kinetic energy α and of momentum β are defined as

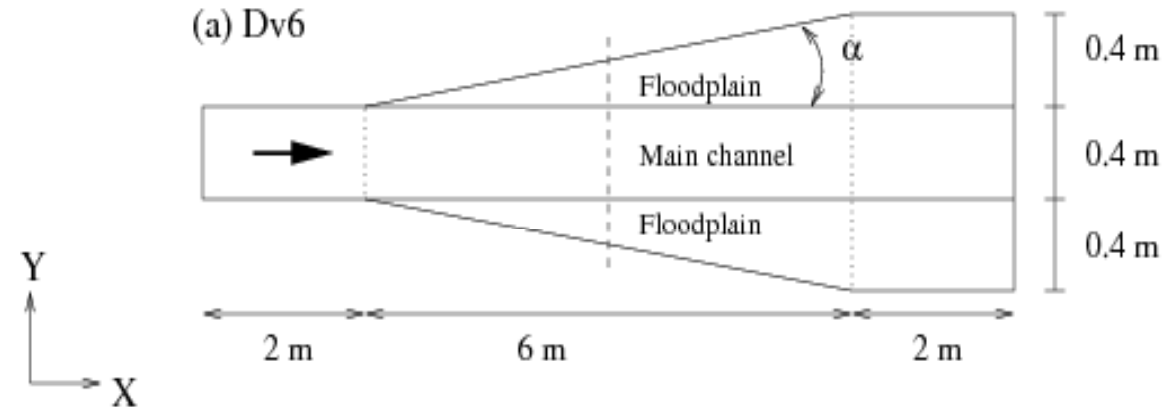
$$\alpha(x)=\frac{\sum_i U_i^3 A_i}{U^3 A}=\frac{\sum_i Q_i^3/A_i^2}{Q^3/A^2} \qquad \beta(x)=\frac{\sum_i U_i^2 A_i}{U^2 A}=\frac{\sum_i Q_i^2/A_i}{Q^2/A}$$

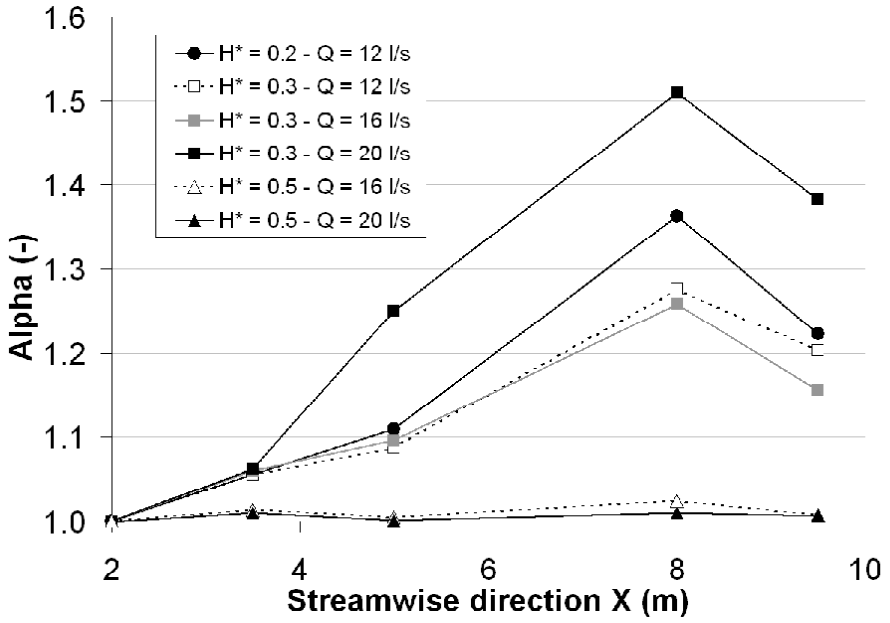
In compound channel, $1 < \alpha \leq 2$ *French (1985)*

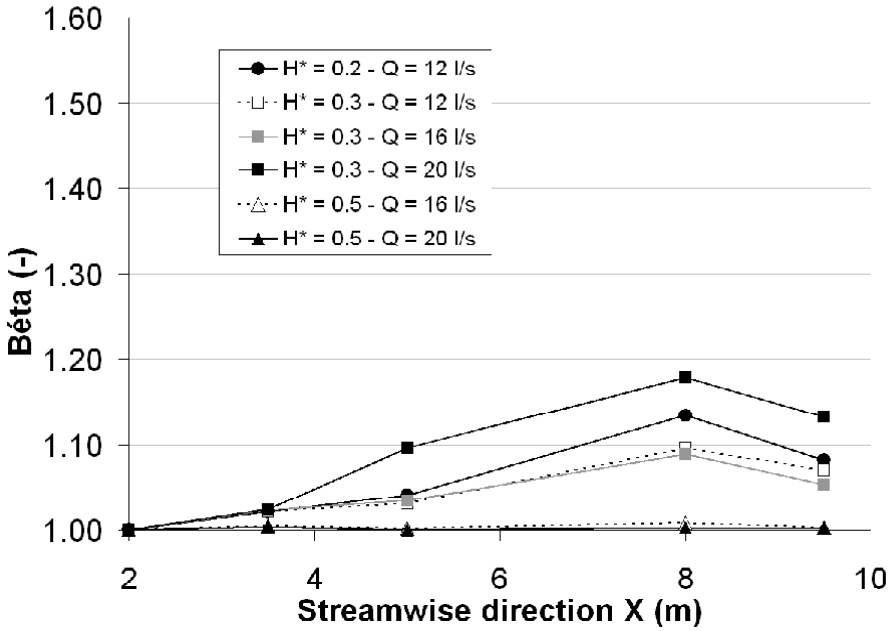
3) 1D models on the total cross-section

UCL, Belgium (10m x 1.2m)

Experimental values of α and β







H* = relative depth

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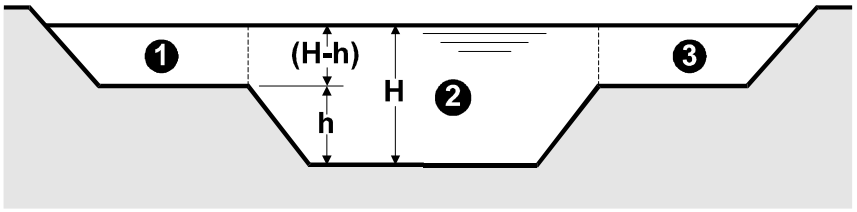
3) 1D total cross-section: link between Q and Q_i

- **Single Channel Method (SCM)**
  - Section as a whole $$Q = \frac{AR^{2/3}}{n} S_0^{1/2}$$
  - Neglects velocity non-uniformity : $R \downarrow$, $Q \downarrow$
- **Divided Channel Method (DCM)**
  - Decomposition in sub-section



$$Q = \Sigma Q_i = \Sigma \frac{A_i R_i^{2/3}}{n_i} S_0^{1/2}$$

  - Neglects momentum transfer : $Q_m \uparrow$, $Q \uparrow$
- **Corrected Divided Channel Method (Debord)**
  - Debord Formula accounts for **turbulence interaction**
  - Empirical correction of Qi, based on measurements in a large flume 60mx3m (EDF, France), *(Nicolet and Uan 1979)*

3) 1D models on the total cross-section

SCM,
Manning 1889
Single Channel Method

Hec-RAS (USCE)

Mike 11 (DHI)

Crue (CNR)

Friction : S_f

(S_a = 0)

Friction : S_f

Turbulence : S^t

(S_a = S_t)

Mage, Rubar3, Talweg-Fluvia (cemagref)

Mascaret (EDF)

Carima (Sogreah), Lido (Cetmef)

EDM,

Axeriv (UCL)

- Friction : S_f

- Turbulence : S^t

-Head loss due to mass exchange: S^m

(S_a = S_t+S_m)

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3) 1D models on the total cross-section

The coupling between water level Z and discharge distribution $\{Q_m, Q_f\}$

Saint-Venant

$$\frac{1}{gA}\frac{d}{dx}\left(\beta A U^2\right)+\frac{dZ}{dx}+S_f=0$$

Bernoulli

$$-\frac{d}{dx}\left(Z+\alpha\frac{U^2}{2g}\right)=S_f+S_a$$

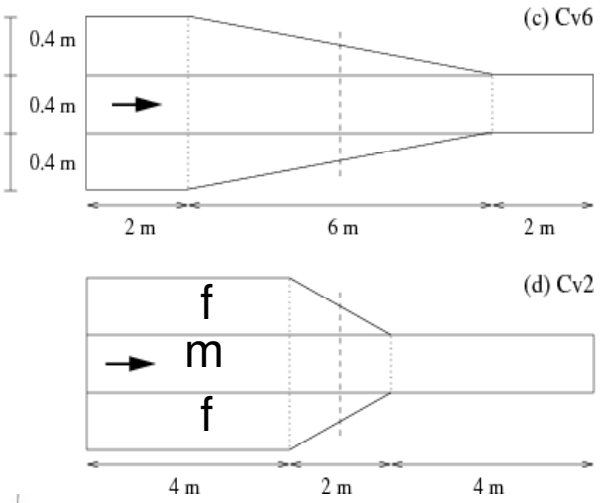
Coupling terms :

- Momentum coeff. = $\beta(Z,Q_m,Q_f)$
- Kinetic Energy coefficient = $\alpha\ (Z, Q_m, Q_f)$
- Friction slope = $S_f\ (Z, Q_m, Q_f)$
- Additional head loss = $S_a\ (Z, Q_m, Q_f)$

3) Link between errors in water level and discharge distribution

Simulations against exp. data

Cv6: $\alpha = 3.8°$



Cv2: $\alpha = 11.2°$

- Flows in symmetrically converging floodplains
- 12 flow cases, with variable
  - Total discharge Q
  - Relative depth H*
- Comparing experimental results with:
  - 1D Saint-Venant (total section)
  - +
  - Corrected DCM

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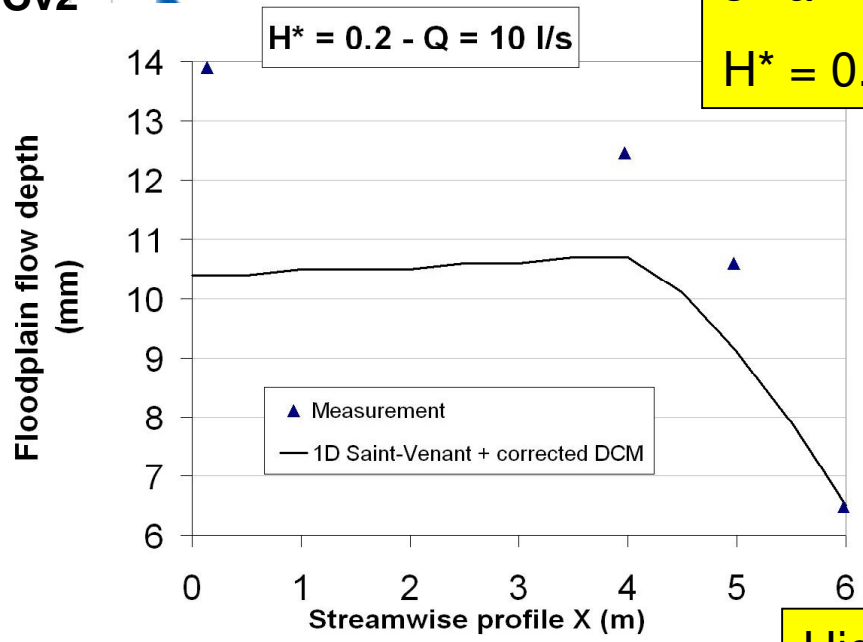
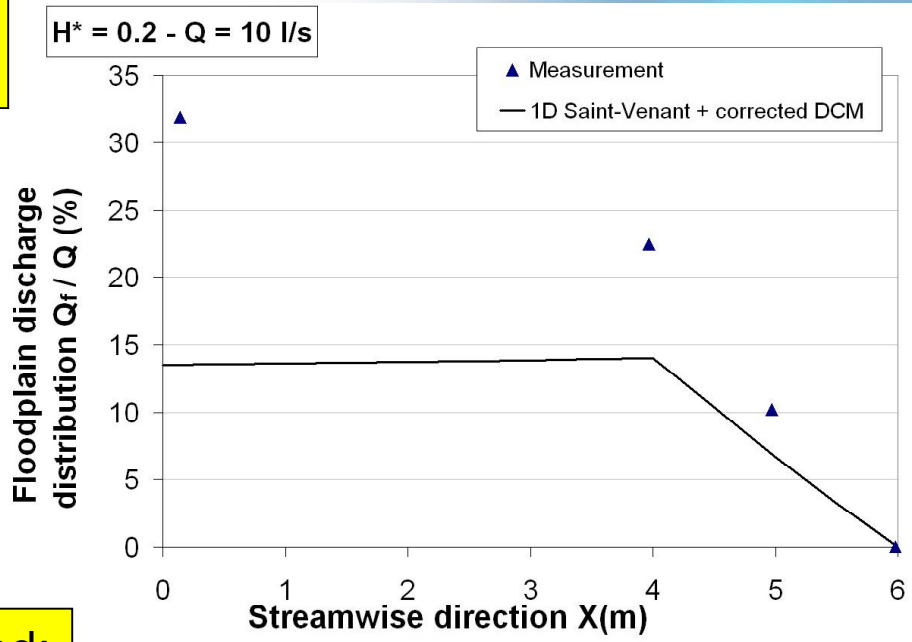
Cv2

small flood:

H* = 0.2

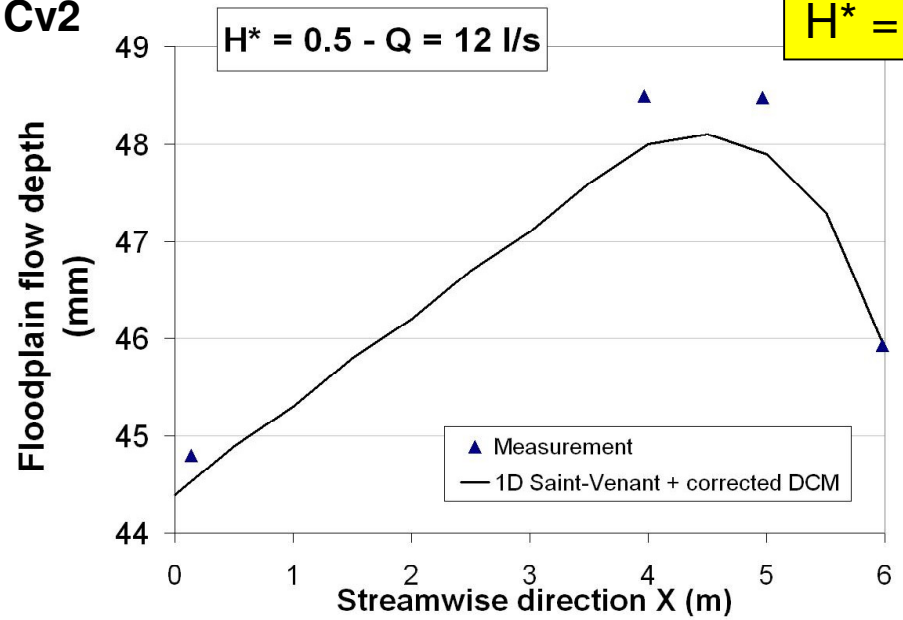


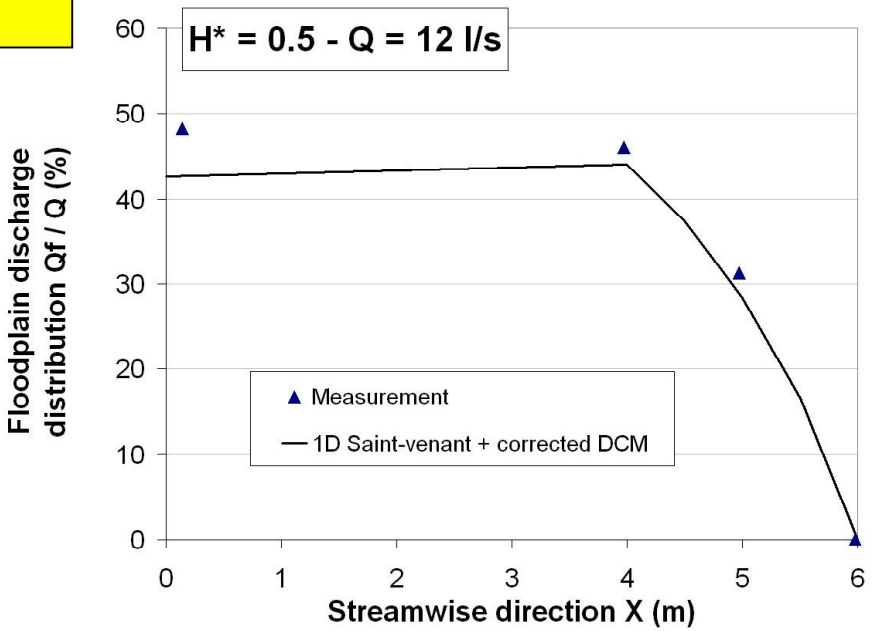


High flood:

H* = 0.5

Cv2





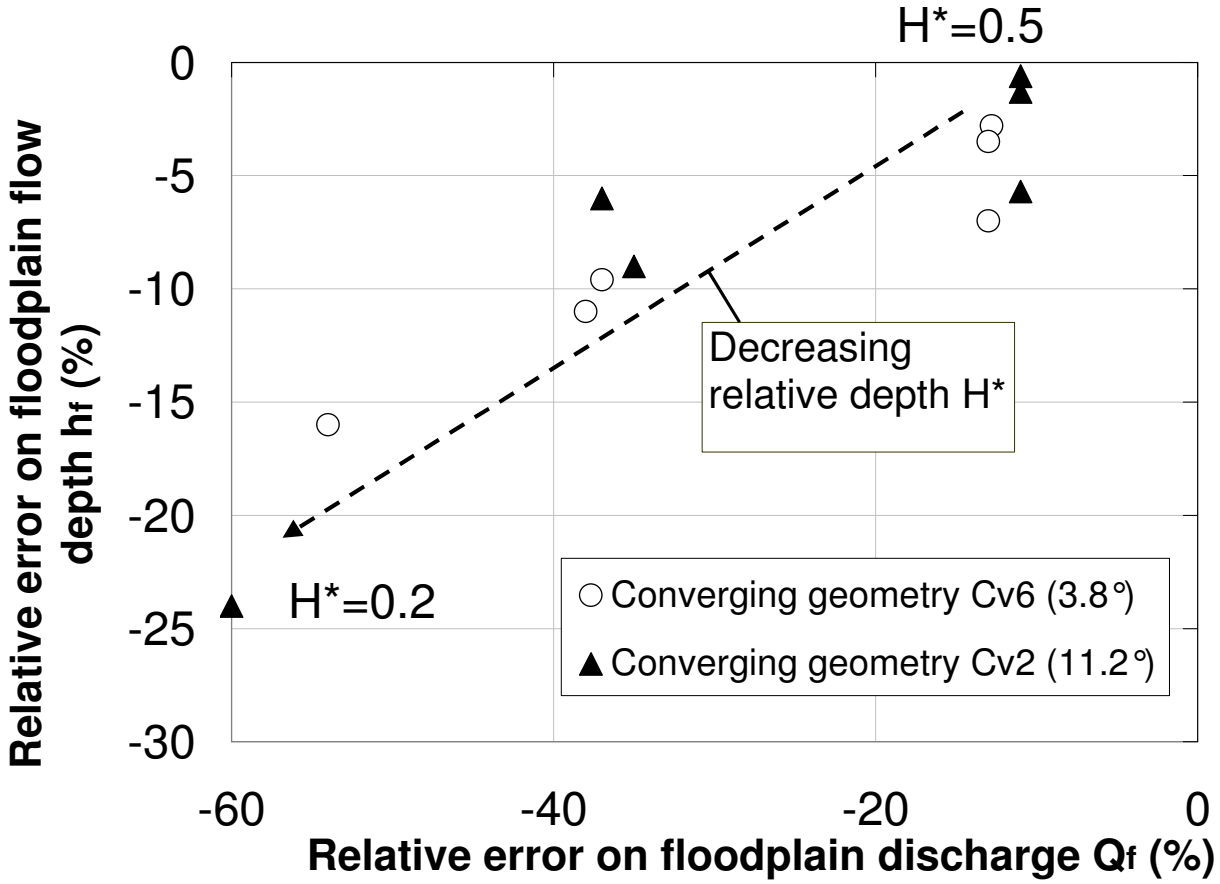
3) Link between errors in flow depth h_f and discharge Q_f

Maximum relative error on h_f and Q_f :

- Max ($hf_{numerical} - hf_{experimental}$)/ $hf_{experimental}$
- Max ($Qf_{numerical} - Qf_{experimental}$)/$Qf_{experimental}$

In *this* geometry:

A clear link between the 2 errors



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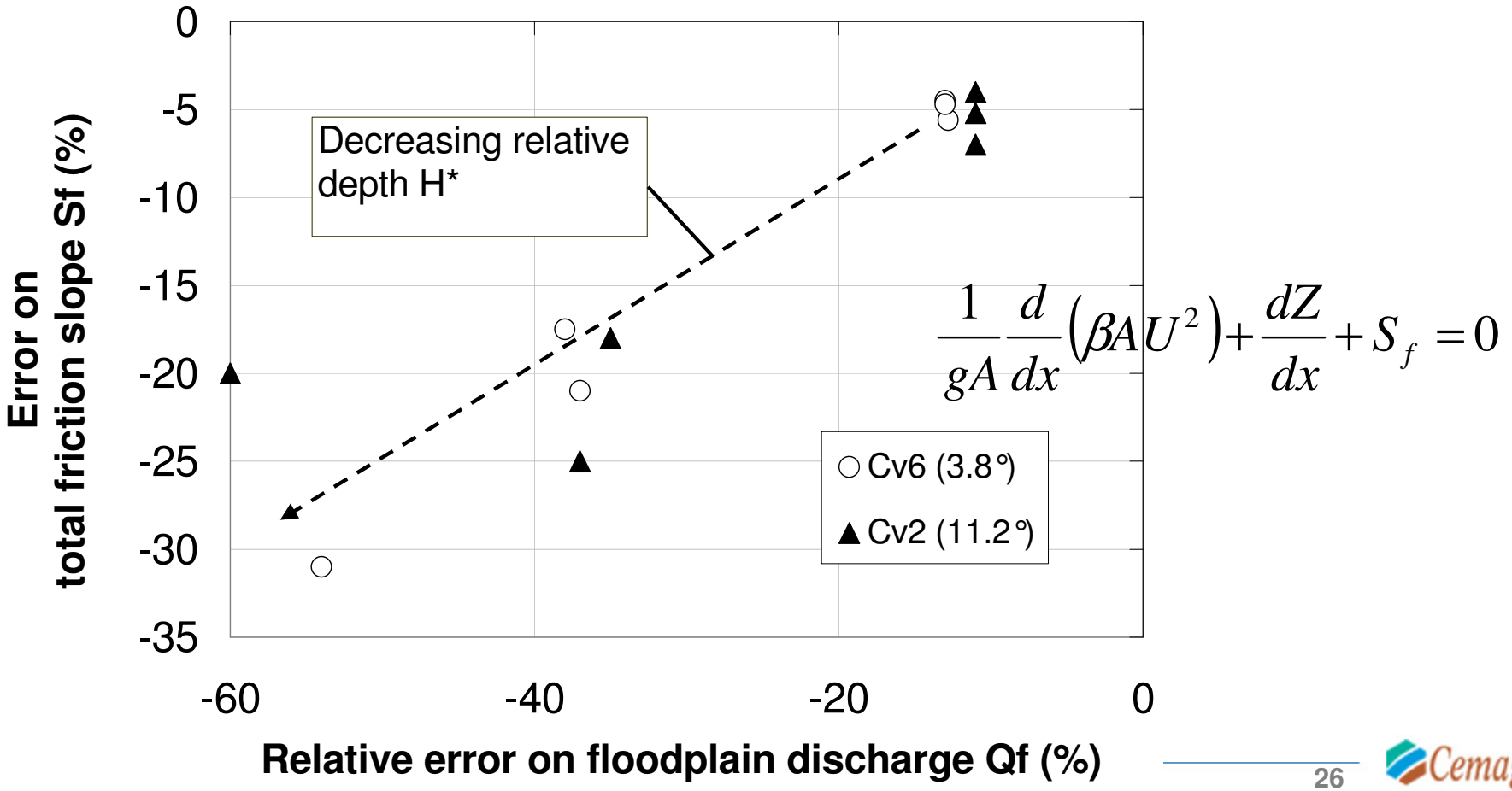
3) Link between errors in flow depth h_f and discharge Q_f

⇒ **link through friction slope S_f on the total cross section**

Max. relative error on friction slope S_f =

Max ($Sf_{numerical}$ – $Sf_{experimental}$)/ $Sf_{experimental}$



$$\frac{1}{gA}\frac{d}{dx}\left(\beta A U^2\right)+\frac{dZ}{dx}+S_f=0$$

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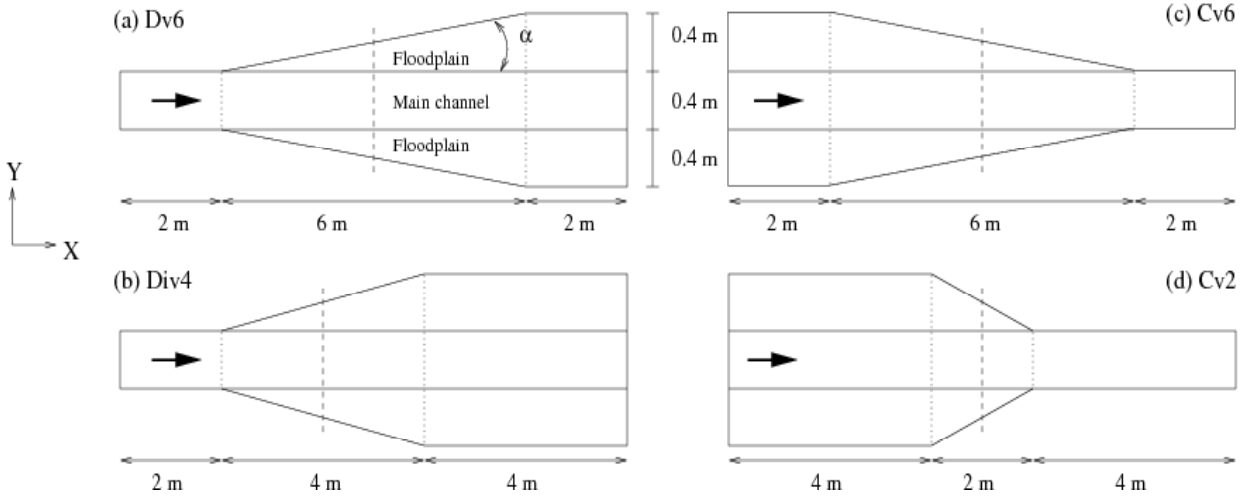
4) The validity of calibrating Manning roughness

n_{mc} = 0.0107 (K_{mc} = 93)

n_{fp} = 0.0107 (K_{fp} = 93)

Dv6: α = 3.8°

Cv6: α = 3.8°



Dv4: α = 5.7°

Cv2: α = 11.2°

Experimental values, calibrated by separating the MC and the FP (single section)

24 experimental flow cases

Notations:

Theses values are denoted: **n_{fp} exp.** and **n_{mc} exp.**

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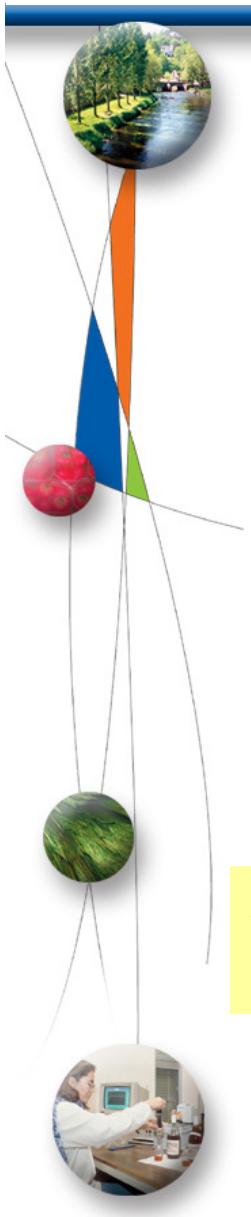
4) The validity of calibrating Manning roughness



Simulation with **HEC-RAS**

-1D Bernoulli Equation

- DCM ($S_a = 0$)

$$-\frac{d}{dx}\left(Z+\alpha\frac{U^2}{2g}\right)=S_f$$

- n_{mc} kept constant = n_{mc} exp. = 0.0107.
- Calibration of n_{fp} to optimize :

  * The water surface profile

  or

  * The discharge distribution (Q_{mc}, Q_{fp})

Scientific issue: using a friction coefficient (Manning) as a flow resistance coefficient ? *Hunter Rouse, (1965)*

$$S_f + S^t + S^m \approx S_f' \ (n_{fp} \text{ optimised}) \ ?$$

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4) The validity of calibrating Manning roughness

Methodology of the optimisation:

Boundary conditions: Upstream total Q ; downstream experimental flow depth.

denoted σ usually

Minimization of standard deviation Σ all along the flow

- On floodplain flow depth h_{fp} $$\sum_{hfp} = \frac{\sqrt{\sum (hfp_{\exp}(i) - hfp_{calc}(i))^2 / n}}{\sum_{i=1}^{n} hfp_{\exp}(i) / n} * 100$$
- On floodplain discharge Q_{fp} $$\sum_{Qfp} = \frac{\sqrt{\sum (Qfp_{\exp}(i) - Qfp_{calc}(i))^2 / n}}{\sum_{i=1}^{n} Qfp_{\exp}(i) / n} * 100$$

Analysis of the relative maximum error on

$$E\max_{hfp} = \max \frac{(hfp_{\exp}(i) - hfp_{calc}(i))}{hfp_{\exp}(i)}$$

$$E\max_{Qfp} = \max \frac{(Qfp_{\exp}(i) - Qfp_{calc}(i))}{Qfp_{\exp}(i)}$$

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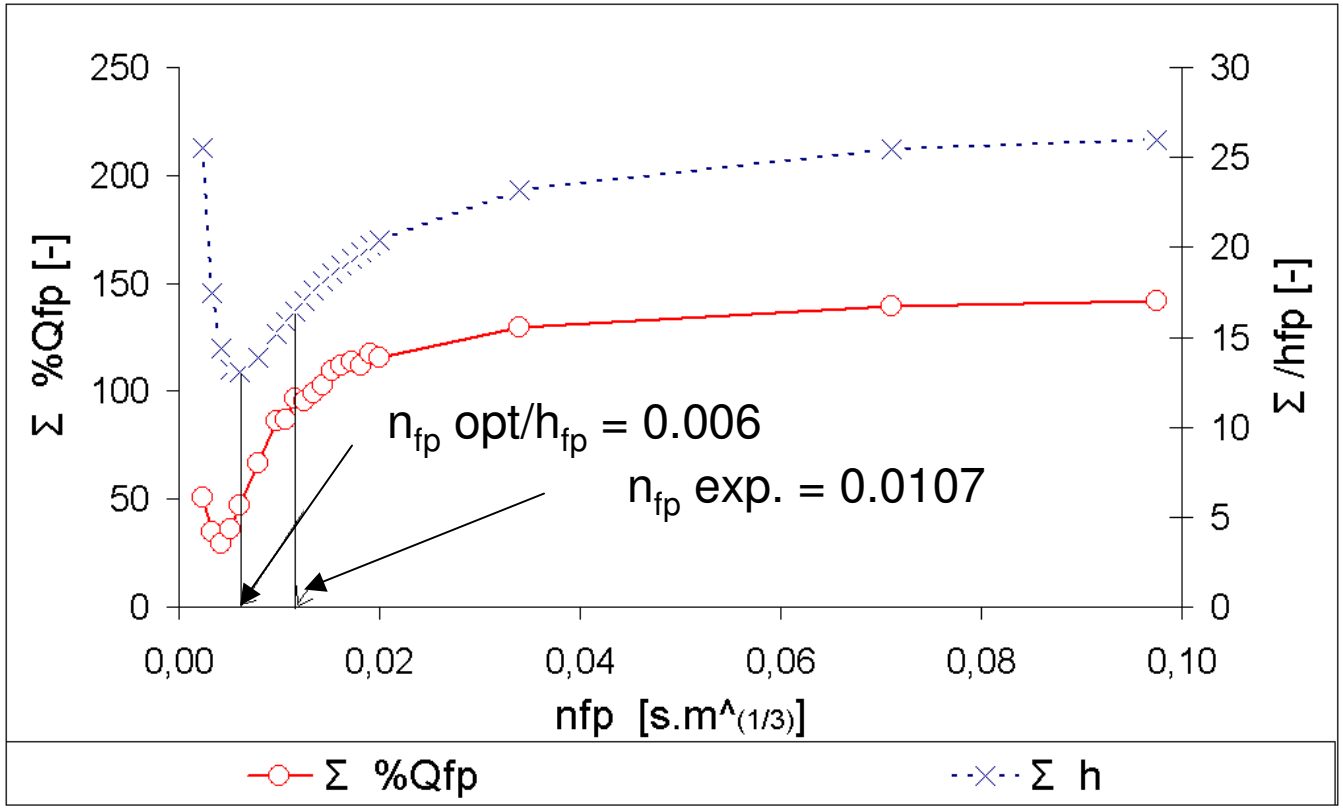
4) The validity of calibrating Manning roughness

Notation: **n_{fp} opt/h_{fp}** = calibrated manning for optimization of depth h_{fp}

n_{fp} opt/Q_{fp} = calibrated manning for optimization of discharge Q_{fp}

- **Cv6, h_r=0,2 (small flood): standard deviation on h_{fp} and Q_{fp}**



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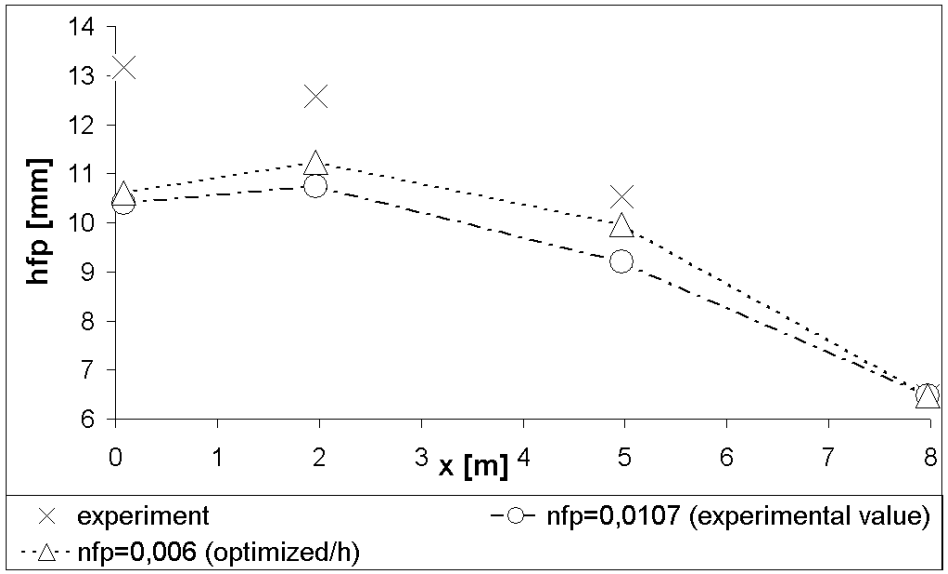
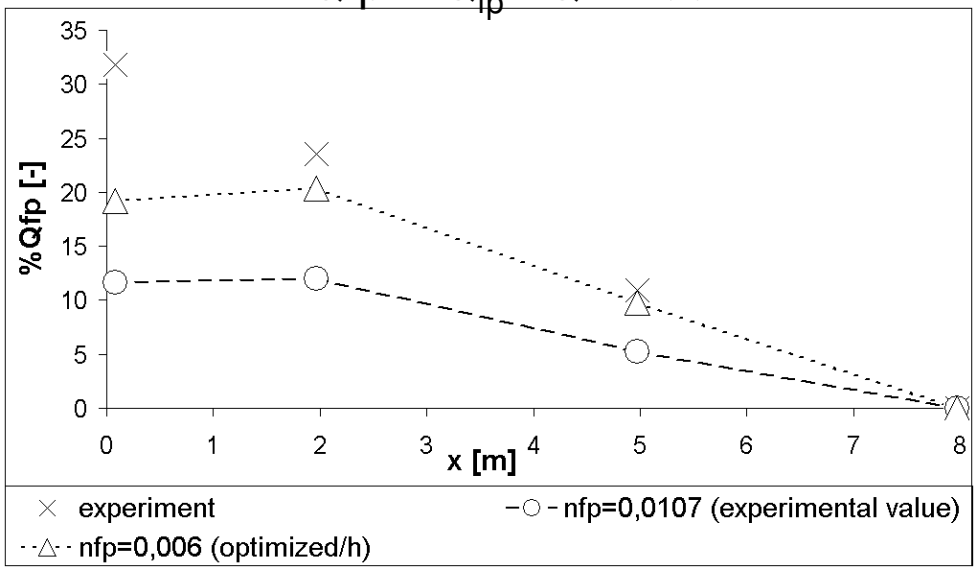
• Cv6, h_r=0,2 (small flood), using n_{fp} opt/h_{fp}

Floodplain flow depth h_{fp}



Discharge ratio in the Floodplain

$\%Qfp = Q_{fp} / Q \times 100$



	$E\max_{hfp}$	$E\max_{\%Qfp}$
nfp_{exp}	**20%**	**63%**
$nfp_{opt/h}$	**19%**	**39%**

??

improvement

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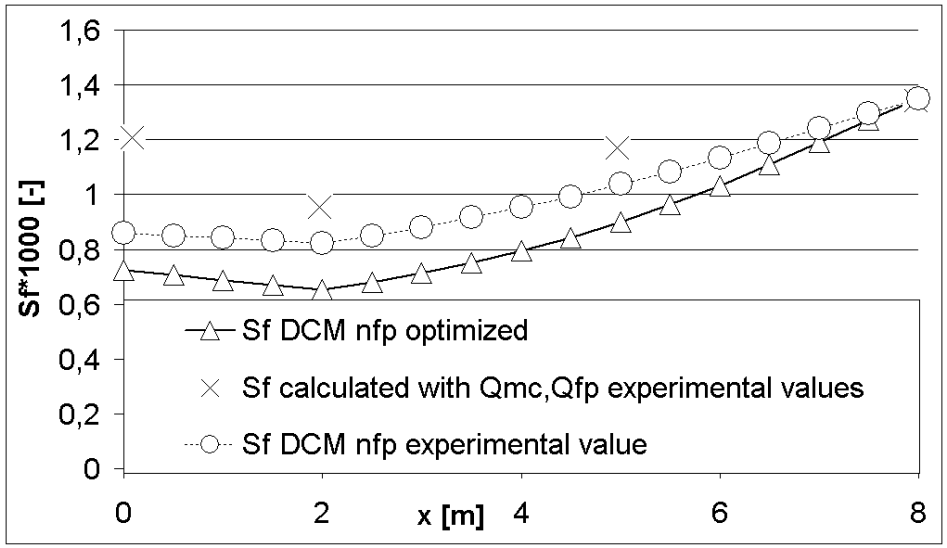
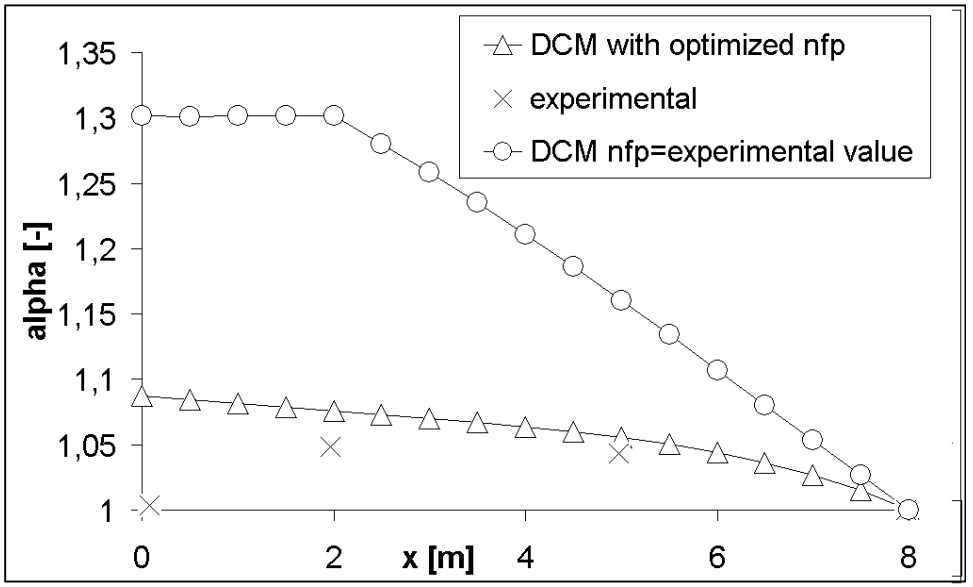
• Cv6, h_r=0,2 (small flood), using n_{fp} opt/h_{fp}

- **HEC-RAS: Bernoulli+DCM:** $$-\frac{d}{dx}\left(Z+\alpha\frac{U^2}{2g}\right)=S_f$$

- **Sf DCM** $$S_f = S_{fi} = \frac{Q^2}{(\sum_i \frac{1}{n_i} A_i R_i^{2/3})^2}$$

Alpha $$\alpha(x)=\frac{Q_{mc}^3 / A_{mc}^{\ 2} + Q_{fp}^3 / A_{fp}^{\ 2}}{Q^3 / A^2}$$





	Sf	alpha
$E\max$ with $nfp_{\exp}$	**29%**	**30%**
$E\max$ with $nfp_{opt/h}$	**40%**	**8%**

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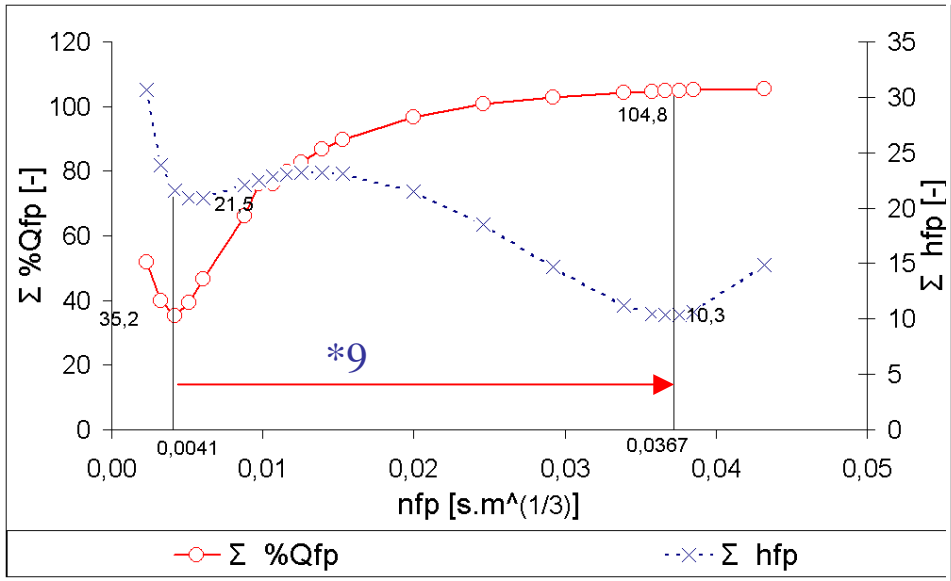
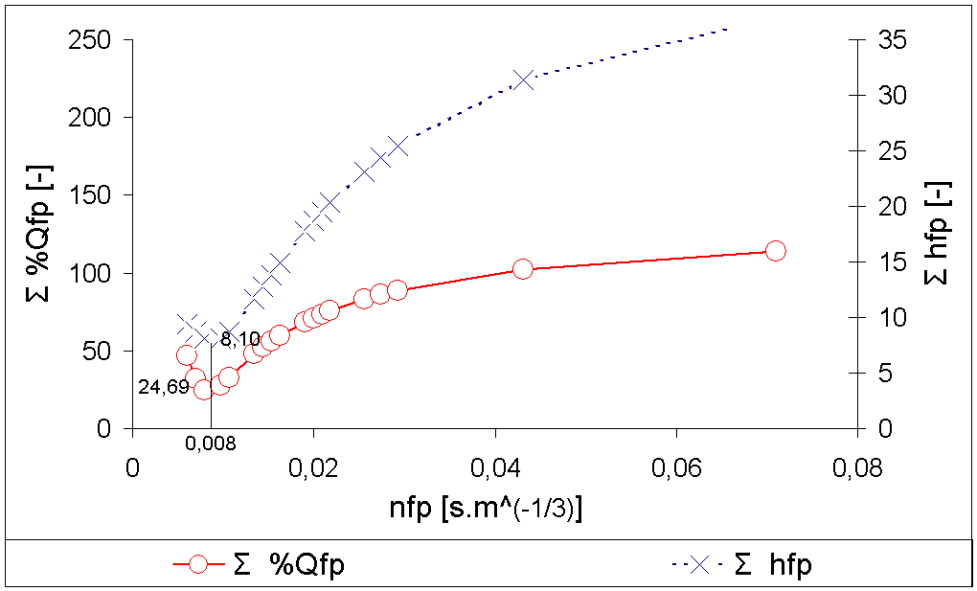
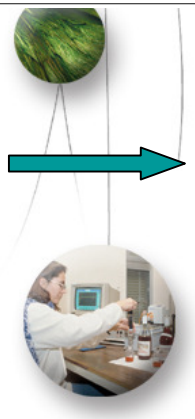
Comparing the calibration of n_{fp} opt/h_{fp} and of n_{fp} opt/Q_{fp}

Cv2 hr=0,2 Q=10L/s



Dv6 hr =0,3 Q=16L/s





Difficulties to simultaneously optimize h_{fp} and Q_{fp} **: factor 9 in Cv2.**

→ The physics of the flow is not well modelled

$$Sf' \neq Sf + St + Sm$$

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Calibration of n_{fp} opt/h_{fp} for the small floods (**hr = 0.2**)

Coated plywood

	$nfp_{opt/h}$	nfp_{exp}	$(nfp_{opt/h} - nf_{exp}) / nf_{exp} * 100$
Cv6 (hr=0,2 Q=12l/s)	0,006	**0,0107**	-44%
Cv2 (hr=0,2 Q=12l/s)	0,038	**0,0107**	+255%
Div6 (hr=0,2 Q=10l/s)	0,0142	**0,0107**	+32%
Div4(hr=0,2 Q=10l/s)	0,006	**0,0107**	-44%
Cv Brusque (hr = 0.2)	0,006	0,0132	-54%

For the same $n_{fp.\ Exp}$:

- **factor 6** for the values of $n_{fp\_optimisé/h}$ in the four geometries with h_r=0,2.
- $(nfp_{opt/h} - nf_{exp}) / nf_{exp} * 100 \in [-44\%; 255\%]$

nfp_{exp}

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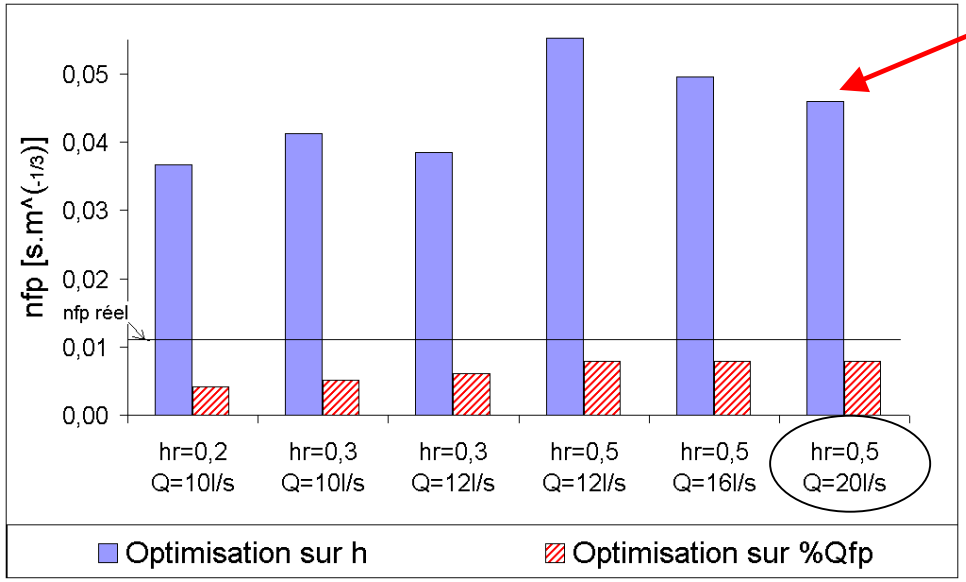
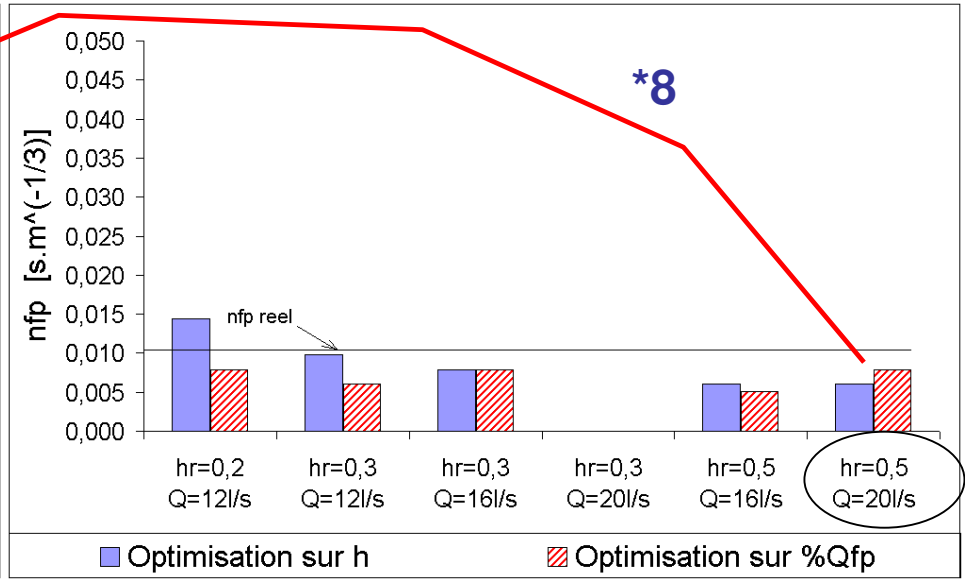
Calibration of n_{fp} opt/h_{fp} for each flow case {Q; h_r}

Cv2 (11.2°)



Cv6 (3.8°)



Factor 8 for values of $n_{fp\_opt/h}$ for the same flood event (Q, h_r)

in 2 various geometries with the same n_{fp}.exp

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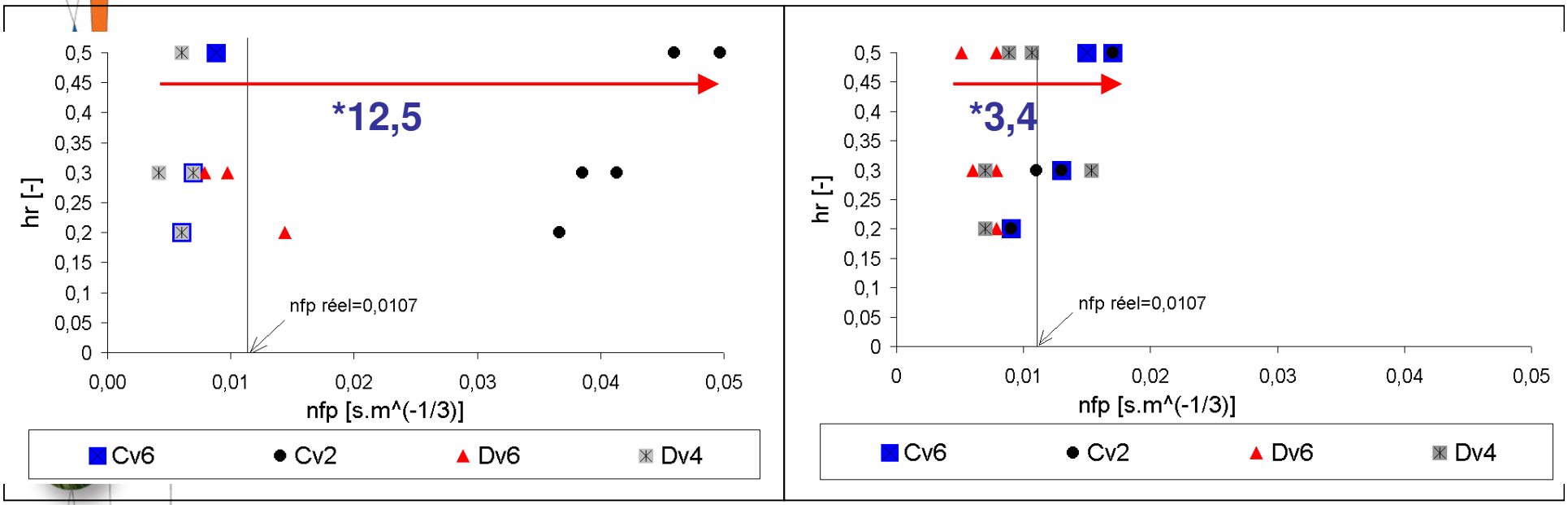
Calibration for **each flow case {Q; h_r}**

using n_{fp}_opt./h_{fp}

using n_{fp}_opt./Q_{fp}



Scattering of n_{fp} → $n_{fp\_opt/h}$ =[0,004;0,0497] : **factor 12,5**

→ $n_{fp\_opt/\%Qfp}$ =[0,005;0,017] : **factor 3,4**

Depending on the geometry and the optimised parameter (h_{fp} or Q_{fp})

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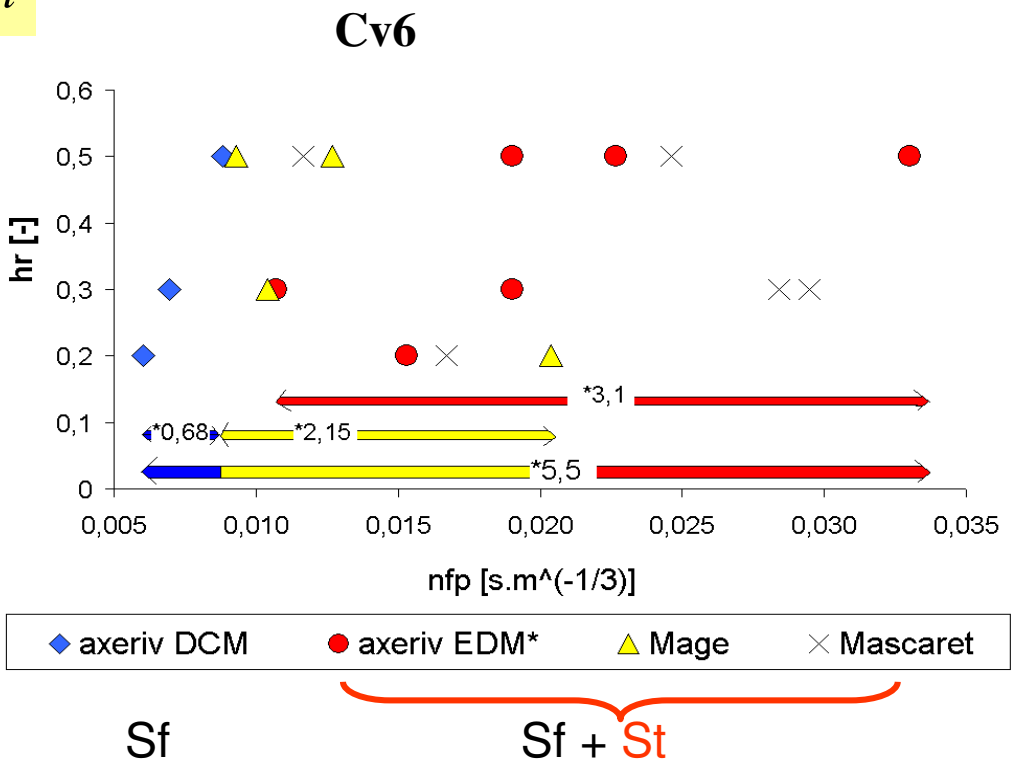
Comparing **four 1D codes on the total cross-section**

$n_{fp\ optimised}\ /\ h$



- 1 code in 1 geometry (e.g. Axeriv EDM*): scattering = factor 3.1
- All 1D codes in 1 geometry : scattering = factor 5.5

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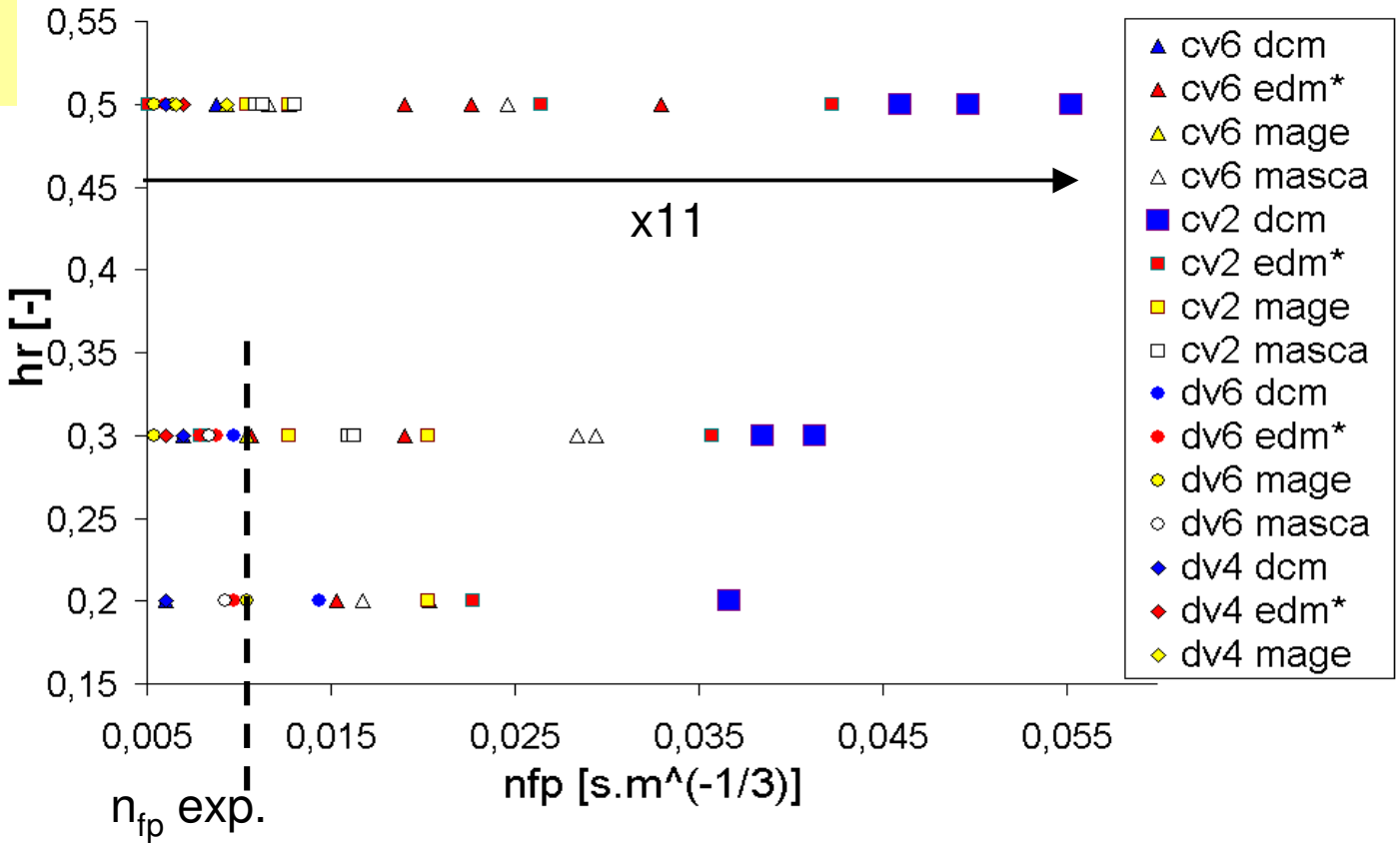
Comparing **four 1D codes in all geometries with same n fp exp**

$n_{fp\ optimised\ /\ h}$



Overall spreading of $\mathbf{n_{fp\_opt/h}}$: **Factor 11,**

(four geometries, four 1D codes, 24 flow cases)

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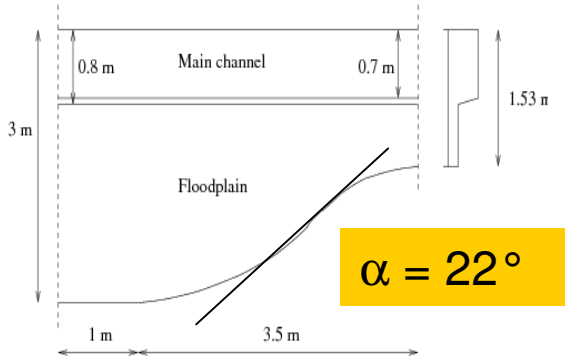
5) The important issues in 1D modelling

Using experimental

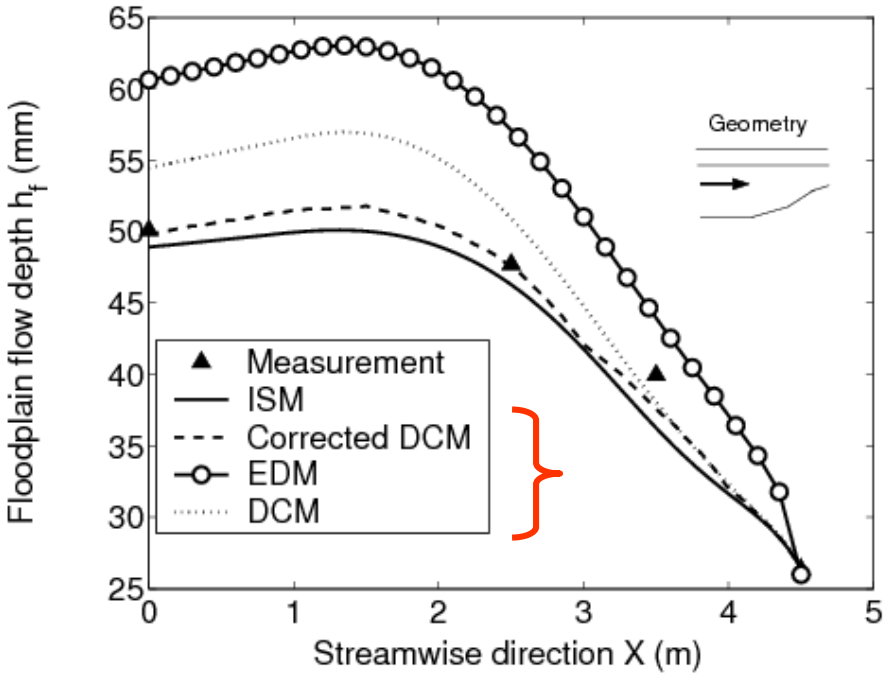
n_{mc} and n_{fp}



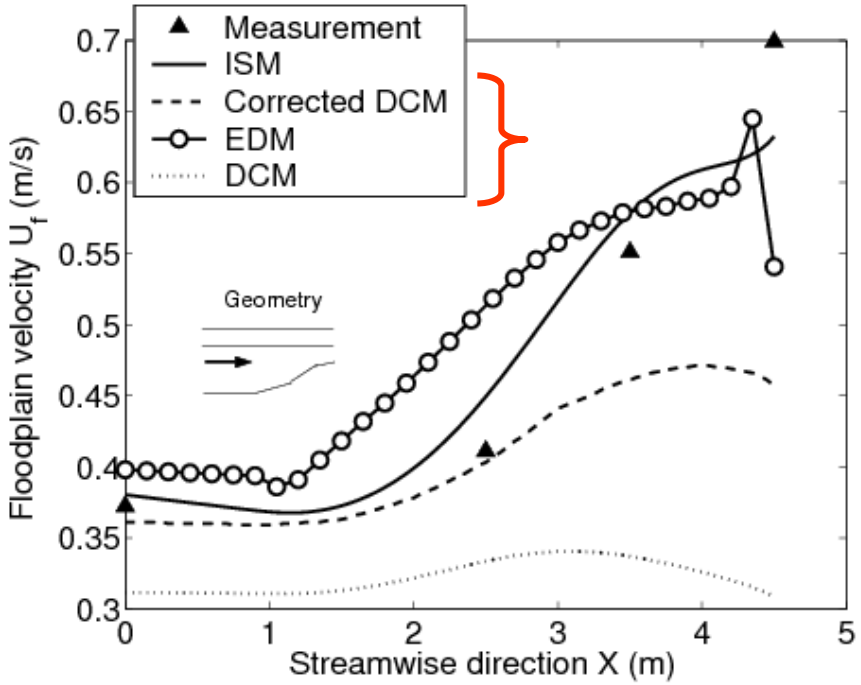
$\alpha = 22°$

Proust et al. (2006), Journal of Hydraulic Engineering

(a) Floodplain flow depth h_f



(b) Floodplain mean velocity U_f

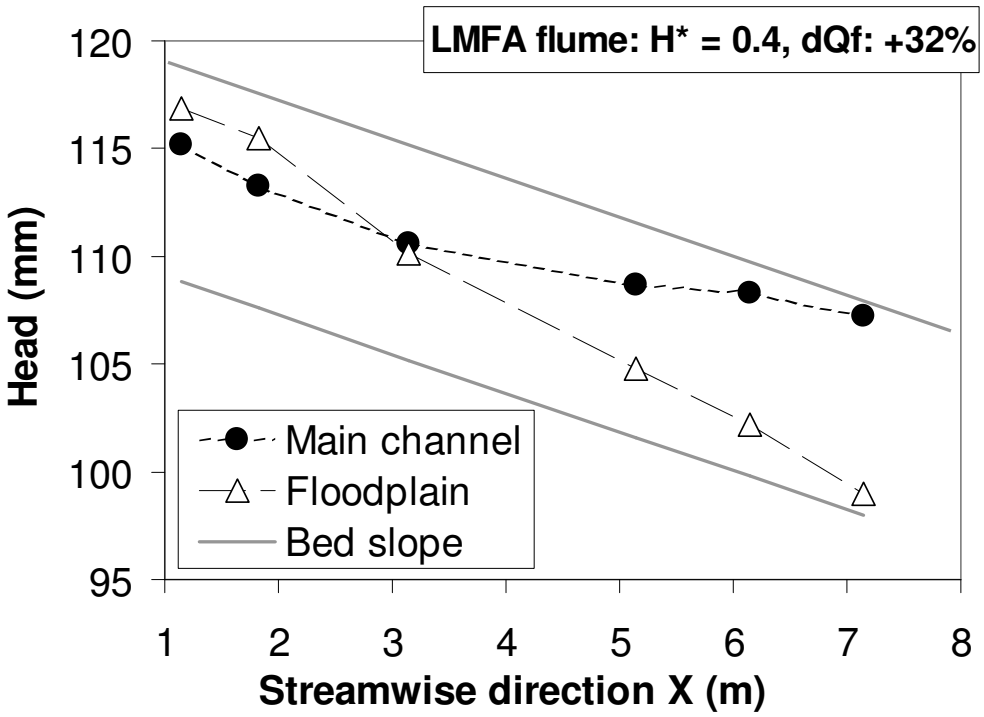


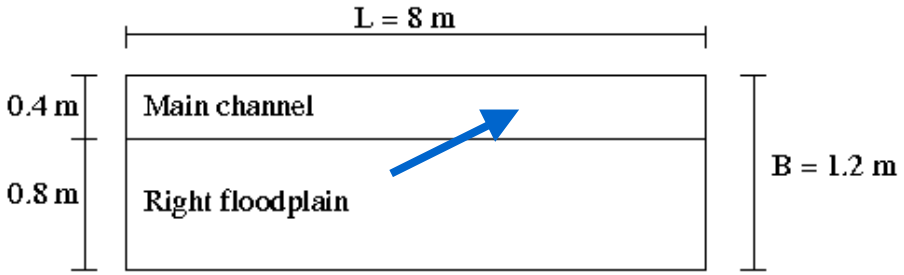
5) The important issues in 1D modelling

Table 2: Respective assumptions of the corrected DCM, EDM and ISM

Assumptions on	Corrected DCM	EDM	ISM
Water surface profile computed on	the **whole cross-section**	the **whole cross-section**	**each subsection**
Bed friction slope S_f computed on the **whole cross-section**	yes	yes, using S_f computed by DCM	–
Bed friction slope S_{fi} computed in **each subsection *i***	yes	yes	yes
Vertical shear stress τ_{ij} between two adjacent subsections i and j	empirical formulas (see e.g. Nicollet and Uan (1979), Ackers (1993))	$\lvert\tau_{ij}\rvert = \rho\Psi^t(U_i - U_j)^2$	
Lateral mass discharge q	–	$\lvert q\rvert = \psi^g \lvert dQ_f/dx\rvert$ with $0 \le \psi^g \le 1$	$\lvert q\rvert = \lvert dQ_f/dx\rvert$
Interfacial velocity U_{int}	–	partially via ψ^g	empirical formulas
Subsection head loss gradient S_{Hi}	$S_{Hm} = S_{Hf}$	$S_{Hm} = S_{Hf}$	**independent**
Upstream discharge distribution	–	–	actual Q_m and Q_f
Downstream discharge distribution (Q_m and Q_f)	implicitly imposed (uniform flow conditions)	implicitly imposed (uniform flow conditions)	–

5) Equality between head loss gradient SH_{fp} and SH_{mc} ?





Erroneous

for non-uniform flow

in both prismatic and non-prismatic geometry.

Cemagref

6) The Independent Sub-sections Method (ISM)

Yen et al. (1985), Yen (1984)

EDM of Bousmar & Zech (1999)

→ ISM (Proust 2005, PhD-Thesis)

ISM: system of 4 ODE that computes $\{h_{fp}; U_{lfp}; U_{mc}; U_{rfp}\}$ simultaneously

ISM	EDM
Water profile on **each subsection**	Water profile on the **whole cross-section**
S_f computed in the **subsections** only.	S_f (whole cross-section) = S_f (DCM)
Independence of head loss gradients : $S_{Hmc} \neq S_{Hfp}$	Equality of head loss gradients: **$S_{Hmc} = S_{Hfp}$**
Measured Q_{mc} and Q_{fp} at the downstream or upstream boundary	Assumes **uniform flow conditions for Q_{mc} and Q_{fp}** at the downstream boundary

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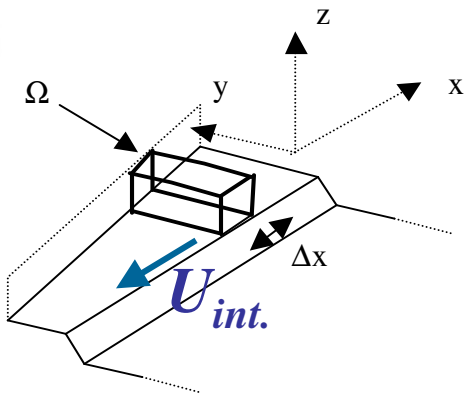
6)The Independent Sub-sections Method (ISM)



Mass continuity in a sub-volume

+

Momentum conservation in a sub-volume

Water profile equation in a subsection

$$\left(1-\frac{U_i^2}{gh_i}\right)\frac{dh_i}{\partial x} = S_o - S_{fi} + \frac{U_i^2}{gB_i}\frac{dB_i}{dx} + \frac{\tau_{xy}.h_{\text{int.}}}{\rho g A_i} + \frac{q(U_{\text{int.}}-U_i)}{gA_i} - \frac{qU_i}{gA_i}$$

S^t S^m *Ma*

Bed friction

Turbulent exchange

Momentum transfer due to mass exch.

Mass conservation

ISM

3 water profile equations

+ 1 mass conservation on the total cross-section

τ_{xy} : **depth-averaged vertical shear stress**

U_{int} : interfacial velocity

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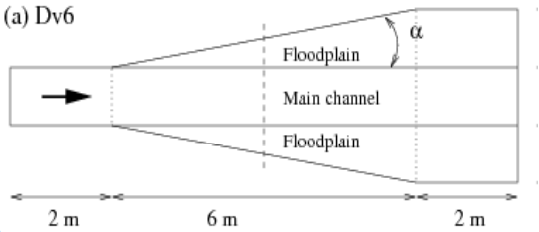
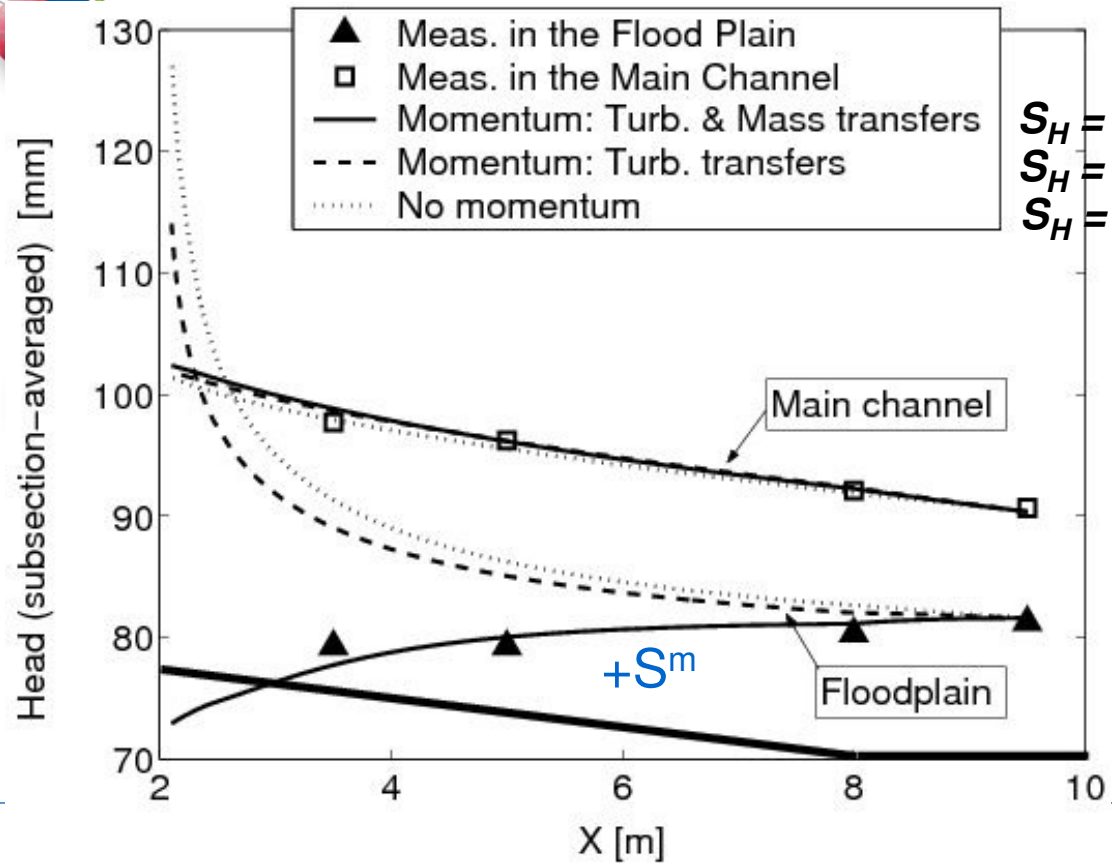
6) ISM :Three types of simulations

- 1) Accounting for the 3 sources of energy loss : $S_H = S_f + S^t + S^m$
- 2) Ignoring S^m at the interfaces: $S_H = S_f + S^t$
- 3) Ignoring S^m and S^t at the interfaces: $S_H = S_f$



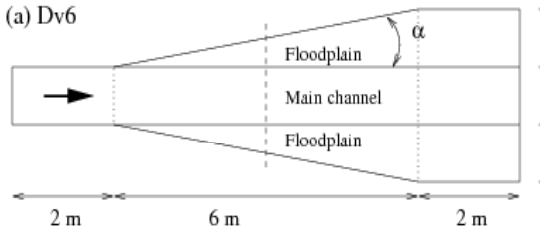
Head (Subsection-averaged) : Dv6 (h_r = 0.3)



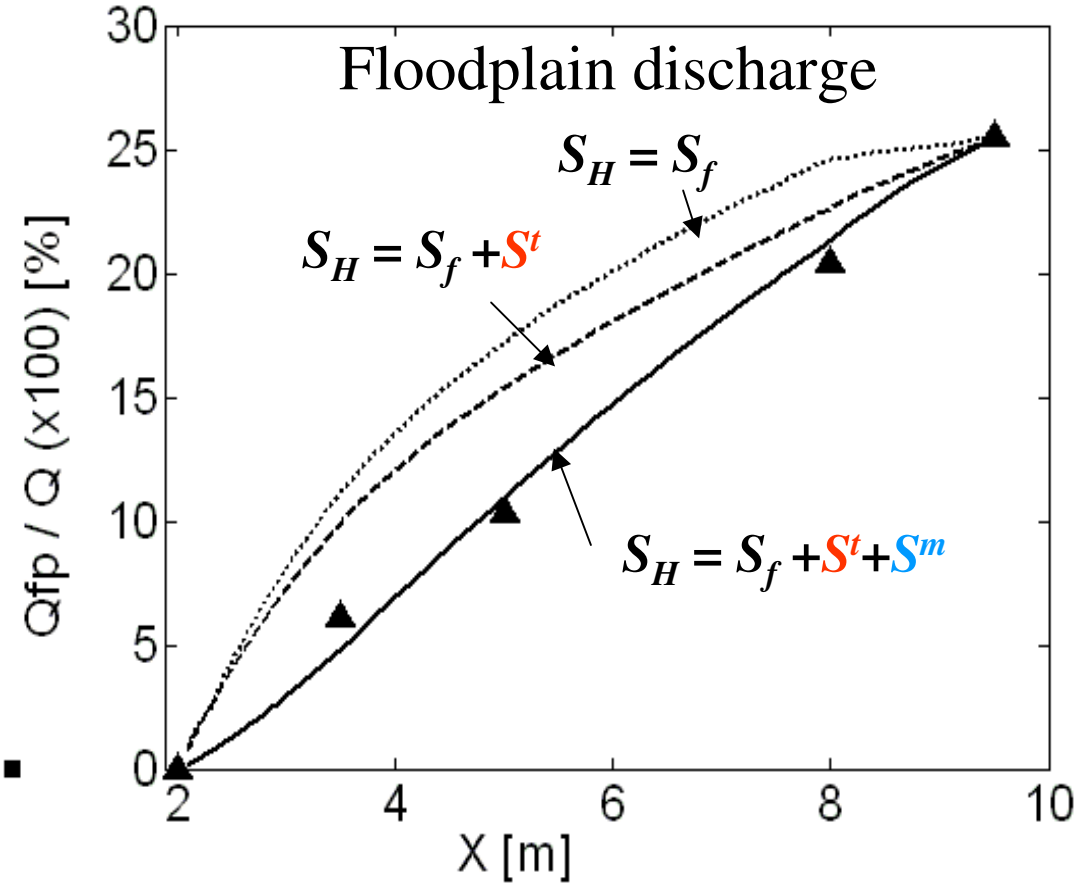
$S_H = S_f + S^t + S^m$
$S_H = S_f + S^t$
$S_H = S_f$

Cemagref

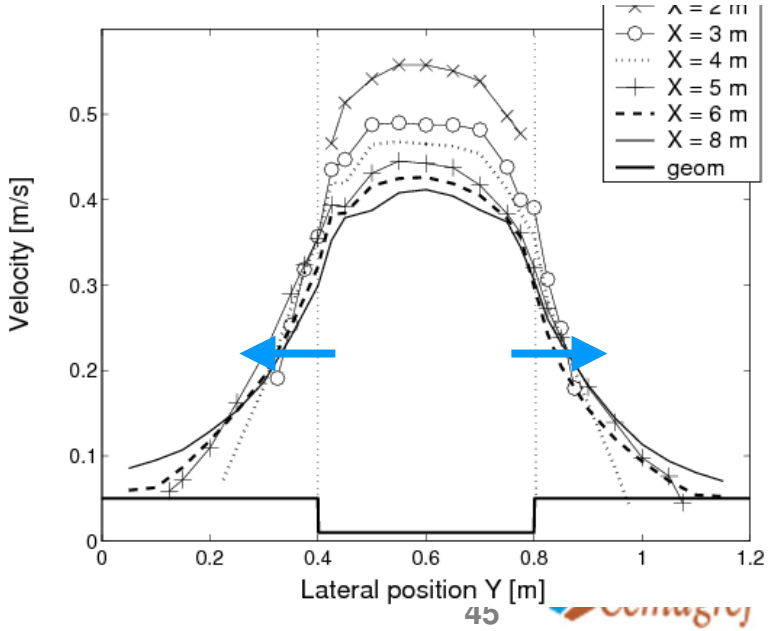
6) ISM :Three types of simulations



Floodplain discharge : Dv6 (h_r = 0.3)



Floodplain flow receives energy from MC, mainly due to mass exchanges **(S^m <0: energy gain)**



7) Conclusion : comparing 1D total section / ISM

Geometries				
Conv. 22°	Conv. 11°	Conv. 3,8°	Diverg. 5,7°	Diverg. 3,8°

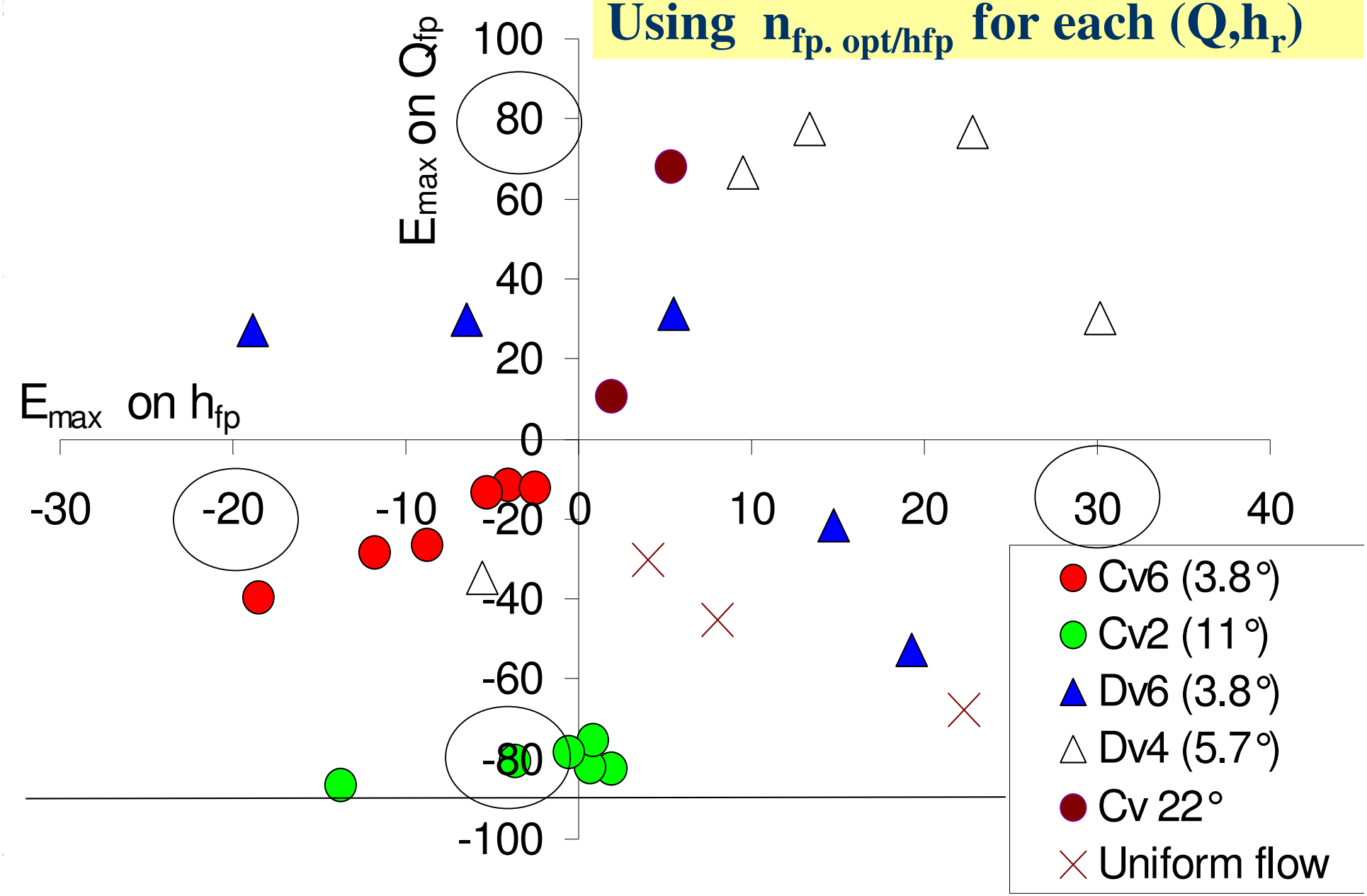
Using n_{fp} exp.

	Error on h_{fp} [%]	Error on Q_{fp} [%]
- *Saint-Venant + Corrected DCM*	[-24 ; +22]	[-60 ; +100]
- *Bernoulli+EDM*	[-22 ; +20]	[-50 ; +133]
-*ISM*	[-8 ; +6]	[-19 ; +16]

Solving the water surface profile in each subsection seems to be efficient.

7) Conclusion : HEC-RAS (Bernoulli + DCM)

Using $n_{fp.\ opt/hfp}$ for each (Q,h_r)



7) Conclusion : HEC-RAS (Bernoulli + DCM)

Recommendation in this context:

- *If field data are available,* calibrating Manning roughness for **small, medium** and **high floods is a minimum**

But using $n_{fp.\ opt/hfp}$ for each (Q,h_r) leads to

Error on h_{fp} [%]	Error on Q_{fp} [%]
[-20 ; +30]	[-86 ; +80]

Both **overestimation** and **underestimation** depending on geometry and (Q, h_r)

Saint-Venant + corrected DCM

Equivalent results were obtained

7) Open discussion