1HD Serre, Boussinesq & Saint-Venant equations

Standard Serre or Green & Naghdi equations

$$\eta_t + (uh)_x = 0$$

Saint-Venant | Boussinesq

$$\underbrace{u_t + uu_x + g\eta_x}_{\text{Saint-Venant}} \underbrace{+ \frac{h}{2}\xi_{xx}u_t - hh_x u_{xt} - \frac{h^2}{3}u_{xxt}}_{\text{Boussinesq}}$$

$$+ \left[h_x\xi_x + (\xi_x)^2\right]u_t - hh_x uu_{xx} + \frac{h^2}{3}u_x u_{xx} - \frac{h^2}{3}uu_{xxx}$$

$$+ \left[h(u_x)^2 + \xi_{xx}u^2\right]\eta_x + \left[h_x\xi_x + (\xi_x)^2 + \frac{3}{2}h\xi_{xx}\right]uu_x$$

$$+ \frac{h}{2}\xi_{xxx}u^2 = 0$$

without restrictions on ε/σ^2

Standard Boussinsq & Saint-Venant equations

$$O(\varepsilon) = O(\sigma^2) \quad \begin{cases} \eta_t + (uh)_x = 0 \\ \underbrace{u_t + uu_x + g\eta_x}_{\text{Saint-Venant}} \underbrace{+ \frac{h_0}{2}\xi_{xx}u_t - h_0 h_x u_{xt} - \frac{h_0^2}{3}u_{xxt}}_{\text{Boussinesq}} = 0 \end{cases}$$

New system of Serre equations,
with improved linear dispersion characteristics

$$\bar{u}_t + g\nabla\eta = 0$$

$$h_t + (uh)_x = 0$$

$$u_t + uu_x + g(h+\xi)_x + (1+\alpha)(\Omega u_t - hh_x u_{xt}) - (1+\beta)\frac{h^2}{3}u_{xxt}$$

$$+\alpha g\Omega(h+\xi)_x - \alpha g h h_x (h+\xi)_{xx} - \beta g\frac{h^2}{3}(h+\xi)_{xxx}$$

$$-hh_x uu_{xx} + \frac{h^2}{3}u_x u_{xx} - \frac{h^2}{3}uu_{xxx} + h(u_x)^2(h+\xi)_x$$

$$+\xi_{xx}u^2(h+\xi)_x + \Omega uu_x + h\xi_{xx}uu_x + \frac{h}{2}\xi_{xxx}u^2 = 0$$

$$\Omega = h_x\xi_x + \frac{1}{2}h\xi_{xx} + (\xi_x)^2$$

Solid phase of the 1HD morphodynamic model

$$(1-p)\xi_t + \langle q_{st} \rangle_x = 0$$

$$\langle q_{st} \rangle = \langle q_{sa} \rangle + \langle q_{ss} \rangle + \langle q_{sk} \rangle + \langle q_{sy} \rangle$$

$$\langle q_{sa} \rangle = \frac{c_a}{g(s-1)} \frac{\varepsilon_a}{\tan\phi} \left(\langle |U|^2 U \rangle - \frac{1}{\tan\phi} \xi_x \langle |U|^3 \rangle \right)$$

$$\langle q_{ss} \rangle = \frac{c_s}{g(s-1)} \frac{\varepsilon_s}{w_q} \left(\langle |U|^3 U \rangle - \frac{\varepsilon_s}{w_q} \xi_x \langle |U|^5 \rangle \right)$$

$$\varepsilon_a = 0.20$$
$$\varepsilon_s = 0.020$$
$$s = 2.0$$
$$d_{50} = 0.001\, m$$

$$\langle q_{sk} \rangle = c_{sk} \left(T_p U_{orb}^2 A_{sk} \right); \quad \langle q_{sy} \rangle = -c_{sy} \left(T_p U_{orb}^2 A_{asy} \right)$$

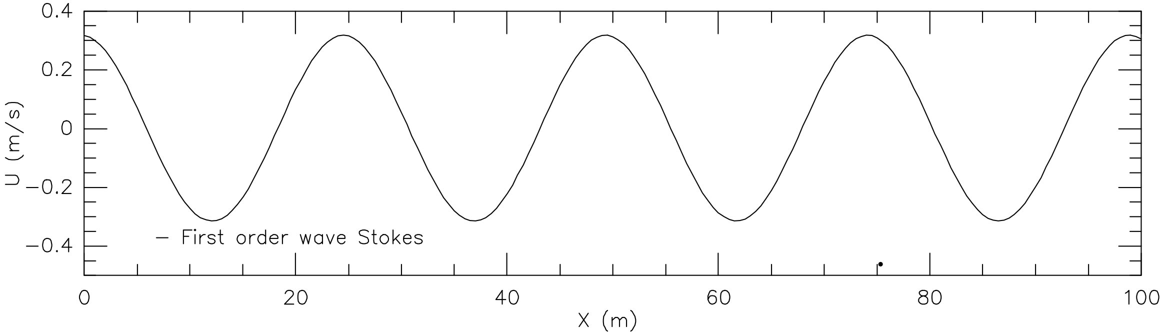
with $U_{orb} = \pi H_{rms} / \left[T_p \sinh(kh) \right]$; $A_{sk} = \langle u^3 \rangle / \langle u^2 \rangle^{3/2}$; $A_{asy} = \langle a^3 \rangle / a_{rms}^3$, $a_{rms} = \langle a^2 \rangle^{1/2}$

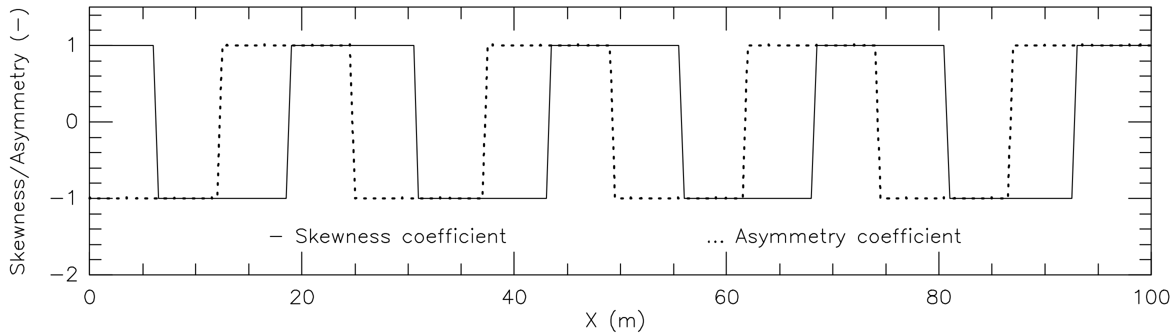
$c_{sk} \approx 5 \times 10^{-6}$ and $c_{sy} \approx 5 \times 10^{-6}$

Stokes 1st order: Skewness & asymmetry of the wave

(Wave period , T = 8.0 s, amplitude, *H* = 0.20 m and wavelength = 24.8 m)

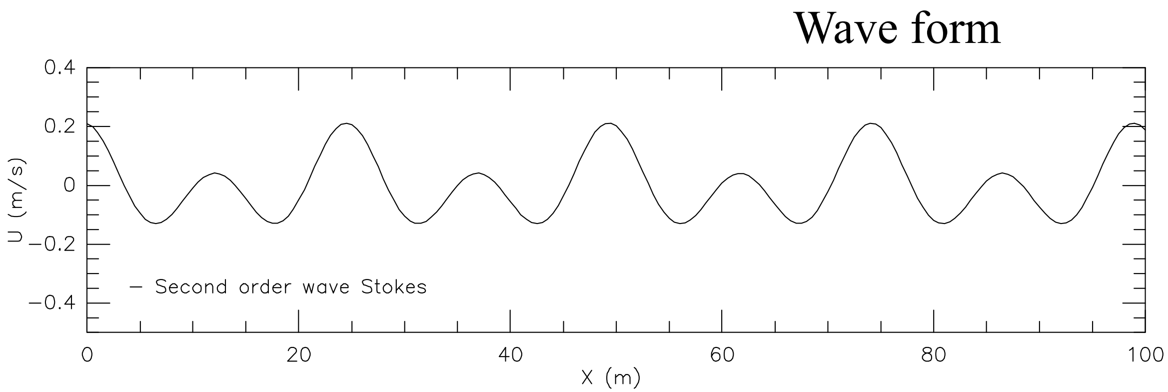
Wave form

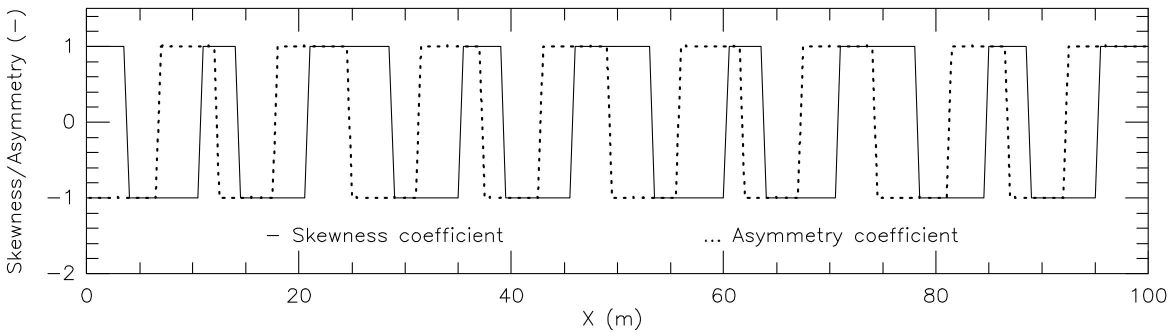




Stokes 2nd order: Skewness & asymmetry of the wave

(Wave period , T = 8.0 s, amplitude, *H* = 0.20 m and wavelength = 24.8 m)





There are three ways to improve our wisdom
[*about vulnerabilities and coastal risks*]:
first, by reflection, which is noblest;
second, by imitation, which is easiest;
and third by experience, which is the bitterest.

(Confucius, *nd*)