BED- AND SUSPENDED-LOAD TRANSPORT

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BED-LOAD TRANSPORT

- *Bed-load* is the mode of transport of sediments where the sediment particles glide, roll or briefly jump, but stay very close to the bed, which they may leave very temporarily
- Limiting values for the separation of different modes of transport

$u_* / w_{ss} \geq 0.1$	bed-load transport	(3.1a)
$u_* / w_{ss} \geq 0.4$	suspended-load transport	(3.1b)

where u_* = shear velocity; and w_{ss} = settling or terminal velocity of particles

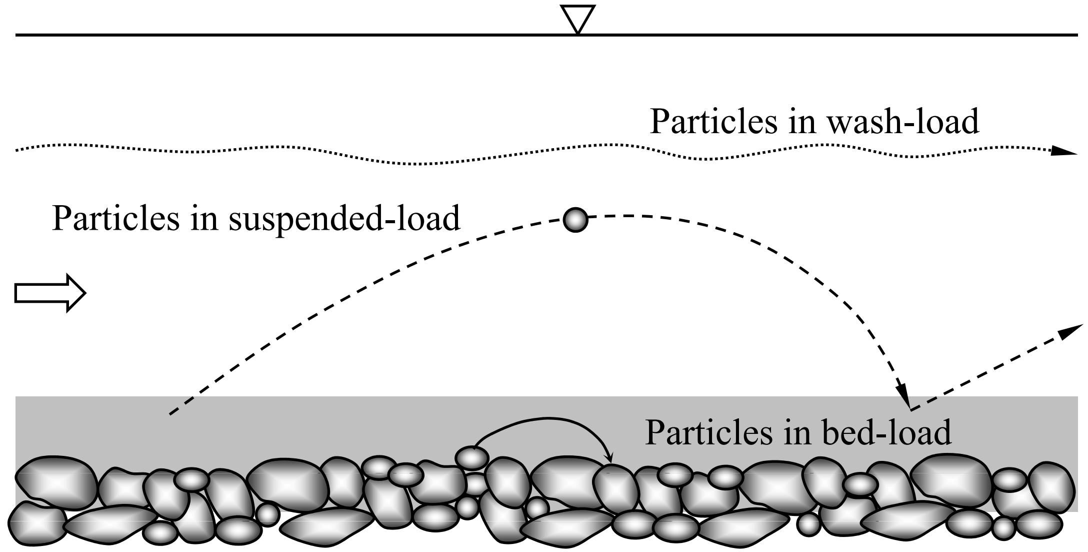

Fig. 3.1 Schematic of different modes of sediment transport

- When the particles stay occasionally in contact with the bed and displace them by making more or less large jumps to remain often surrounded by water, the mode of transport is termed *suspended-load*
- Mode of transport of very fine particles is as *wash-load*

DuBoys' Approach

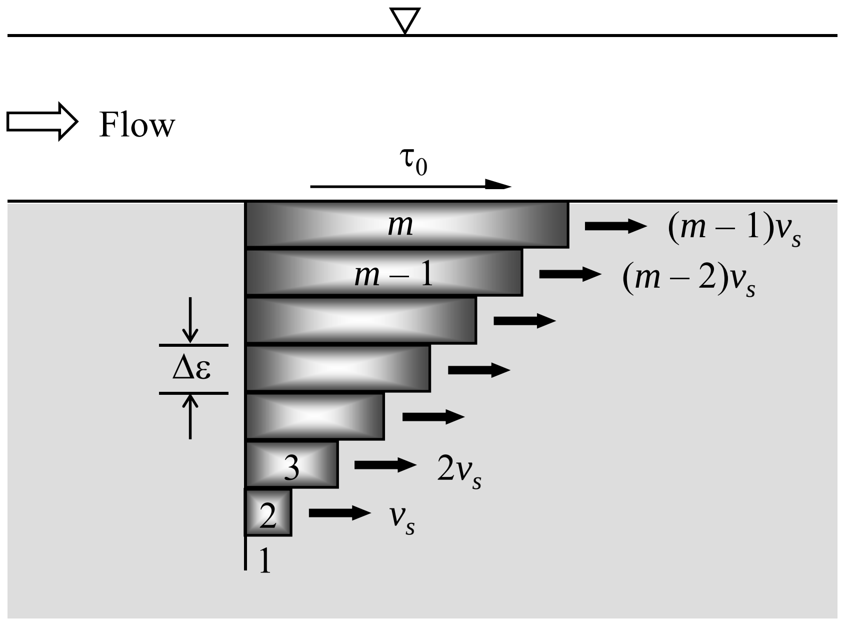

Fig. 3.2 Defination sketch of DuBoys bed-load model

- **DuBoys** (1879) assumed that the sediment moves in layer having a thickness $\Delta\varepsilon$

- The layers move due to the tractive force given by bed shear stress τ_0 (= $\rho g h S$) is applied to them, where ρ = mass density of fluid; g = gravitational acceleration; h = flow depth; and S = streamwise bed slope

- The top layer is one where the tractive force balances the resistance force between these layers

$$\tau_0 = \rho g h S = \mu m \Delta\varepsilon(\rho_s - \rho) g \qquad (3.2)$$

where μ = frictional coefficient; m = number of layers; and ρ_s = mass density of sediment

- The fastest moving layer is the top layer and moves with the velocity of $(m - 1)v_s$
- If the layer between first and m-th moves according to a linear velocity distribution, then the amount of sediment (in volume per unit time per unit width i.e. m^3/sm) is given by

$$q_b = \Delta\varepsilon v_s m(m-1)/2 \qquad (3.3)$$

- The critical condition at which sediment motion is just about to begin is given by $m = 1$

$$\tau_c = \mu\Delta\varepsilon(\rho_s - \rho) g \qquad (3.4)$$

- This, in turn, results in the relationship

$$\tau_0 = m\tau_c \qquad (3.5)$$

- It is introduced into Eq (3.3) and the following is obtained

$$q_b = \frac{\Delta\varepsilon v_s}{2\tau_c}\tau_0(\tau_0 - \tau_c) \qquad (3.6)$$

- **DuBoys** (1879) referred the first term within the square bracket in RHS of Eq. (3.6) as a characteristic of sediment coefficient and gave it a symbol χ

$$q_b = \chi\tau_0(\tau_0 - \tau_c) \qquad (3.7)$$

- **Straub** (see **Rouse** 1950) related χ to the particle size *d* (in SI units) (0.125 mm < *d* < 4 mm) as

$$\chi = 6.89 \times 10^{-6} / d^{0.75} \qquad (3.8)$$

Other Empirical Equations of DuBuys Type:

- **Schoklitsch** (1934) proposed for particle size 0.305 mm < d < 7.02 mm

$$g_b = \frac{7000}{d_{50}^{0.5}} S^{1.5}(q - q_c) \qquad (3.9)$$

where g_b = bed-load transport rate in weight per unit width; q = flow rate per unit width; and q_c = $1.944\times10^{-5}/S^{1.33}$ (m³/sm)

- **Schoklitsch** (1950) later modified the equation for $d \geq 0.6$ mm

$$g_b = 2500 S^{1.5}(q - q_c) \qquad (3.10)$$

where $q_c = h_c^{5/3}S^{1/2}/n = 0.26\Delta^{3/5}d^{3/2}/S^{7/6}$; n = Manning coefficient; h_c = critical flow depth; $\Delta = s - 1$; and s = relative density of sediment

- **Shields** (1936) put forward

$$q_b = 10\frac{qS}{s}(\Theta - \Theta_c) \qquad (3.11)$$

where Θ and Θ_c = Shields and critical Shields parameters, respectively

- The Shields parameter is given by $\Theta = \tau_0/(\Delta\rho g d)$ and Θ_c corresponds to τ_c

$$g_b = 10\frac{\rho g q S}{s}(\Theta - \Theta_c) \tag{3.12}$$

- **Meyer-Peter** (1951) gave the following equation

$$q_b = 8(\Delta g d^3)^{0.5}(\Theta - \Theta_c)^{1.5} \tag{3.13a}$$

$$g_b = 8\rho_s g(\Delta g d^3)^{0.5}(\Theta - \Theta_c)^{1.5} \tag{3.13b}$$

- For gravel-bed rivers, **Parker** (1979) proposes

$$q_b = 11.2(\Delta g d^3)^{0.5}\frac{(\Theta - 0.03)^{4.5}}{\Theta^3} \tag{3.14}$$

- **Nielson**'s (1992) equation for sand and gravels (0.69 mm ≤ d ≤ 28.7 mm)

$$q_b = (\Delta g d^3)^{0.5}\Theta(12\Theta - 0.05) \tag{3.15}$$

- $\Phi = q_b/(\Delta g d^3)^{0.5} = g_b/[(\rho_s g)(\Delta g d^3)^{0.5}] = g_{bs}(s/\Delta)/[(\rho_s g)(\Delta g d^3)^{0.5}]$, where g_{bs} = bed-load transport rate in submerged weight per unit width

Einstein's Bed-Load Function

Einstein (1950) developed a bed-load model from probabilistic concept

Rate of Deposition:

- The average traveling distance L_0 is the distance that a particle travels from its starting point until it is deposited on the bed
- The step length of a particle diameter d can be expressed as λd and for spherical particles, $\lambda = 100$
- If after a particle travels a step length, it falls on the bed at a point where a local lift force is greater than submerged weight of particle, and the particle does not stop moving but travels a second step length
- If p is the probability of the lift force being greater than the submerged weight, $n(1 - p)$ particles deposit on the bed after traveling a step length, where n is the number of particles in motion
- Only np particles continue moving
- After traveling the second step length, $np(1 - p)$ more particles stop moving and only np^2 particles remain in motion

- All n particles stop on the bed after some time elapses
- The traveling distance can be determined as

$$L_0 = \sum_{n=0}^{\infty}(1-p)p^n(n+1)\lambda d = \frac{\lambda d}{1-p} \qquad (3.16)$$

- If g_b represents the rate of bed-load transport in dry weight, than rate of deposition on unit area = $g_b/(L_0 \times 1) = g_b(1 - p)/(\lambda d)$

Rate of Erosion:

- The number of particles per unit area can be estimated as $1/(A_1 d^2)$, and their total weight is $A_2 \rho_s g d^3/(A_1 d^2)$
- If p is the probability for a particle to begin to move, sediment with a total weight of $(A_2 \rho_s g/A_1)pd$ is eroded from the bed per unit time, where A_1 and A_2 are coefficients related to the shape of the particles
- Exchange time or time for a particle to be removed is assumed proportional to the time for a particle to fall a length of one diameter in still water

$$t \sim \frac{d}{w_{ss}} = A_3 (d / \Delta g)^{0.5} \qquad (3.17)$$

where A_3 = constant of time scale

- the rate of erosion per unit area of the bed surface

is $(A_2 \rho_s g / A_1) p d / [A_3 (d/\Delta g)^{0.5}] = \rho_s \Delta^{0.5} g^{1.5} p d^{0.5} [A_2/(A_1 A_3)]$

Equilibrium of Sediment Transport:

- Sediment transport is in equilibrium if the amount of sediment

eroded from the bed is equal to the amount of sediment deposited on the bed for a given time

$$\frac{g_b (1-p)}{\lambda d} = \rho_s \Delta^{0.5} g^{1.5} p d^{0.5} \frac{A_2}{A_1 A_3} \qquad (3.18)$$

- It can be written as

$$\frac{p}{1-p} = A_* \Phi \qquad (3.19)$$

where $A_* = A_1 A_3/(\lambda A_2)$; and Φ =

$q_b/(\rho_s \Delta^{0.5} g^{1.5} d^{1.5})$

- The parameter Φ is called *bed-load transport intensity* and the probability is given by

$$p = \frac{A_* \Phi}{1 + A_* \Phi} \tag{3.20}$$

Probability Determination:

- The submerged weight of particle F_G is

$$F_G = A_2 (\rho_s - \rho) g d^3 \tag{3.21}$$

- The lift force F_L is given by

$$F_L = \frac{1}{2} C_L A_1 d^2 \rho u_b^2 \tag{3.22}$$

where C_L = lift coefficient; and u_b = effective velocity at the edge of the viscous sub-layer

- **Einstein and El-Samni** (1949) found that for uniform sediment, if velocity at an elevation $z = 0.35X$ is taken as effective velocity u_b in Eq. (3.22), the distribution of fluctuating lift force follows normal distribution with a standard deviation equal to half the mean value and $C_L = 0.178$

- The effective velocity u_b is expressed as $u_b/u_* = 5.75 \log[(30.2)(0.35X/\Delta_k)]$, where $X(\Delta_k/\delta > 1.8) = 0.77\Delta_k$; $X(\Delta_k/\delta < 1.8) = 1.39\delta$; Δ_k = apparent roughness (= k_s/x); and δ = viscous sub-layer thickness (= $11.6\nu/u_*$)
- The apparent roughness Δ_k can be obtained from the curve given by **Einstein** (1950) (Fig. 3.3)

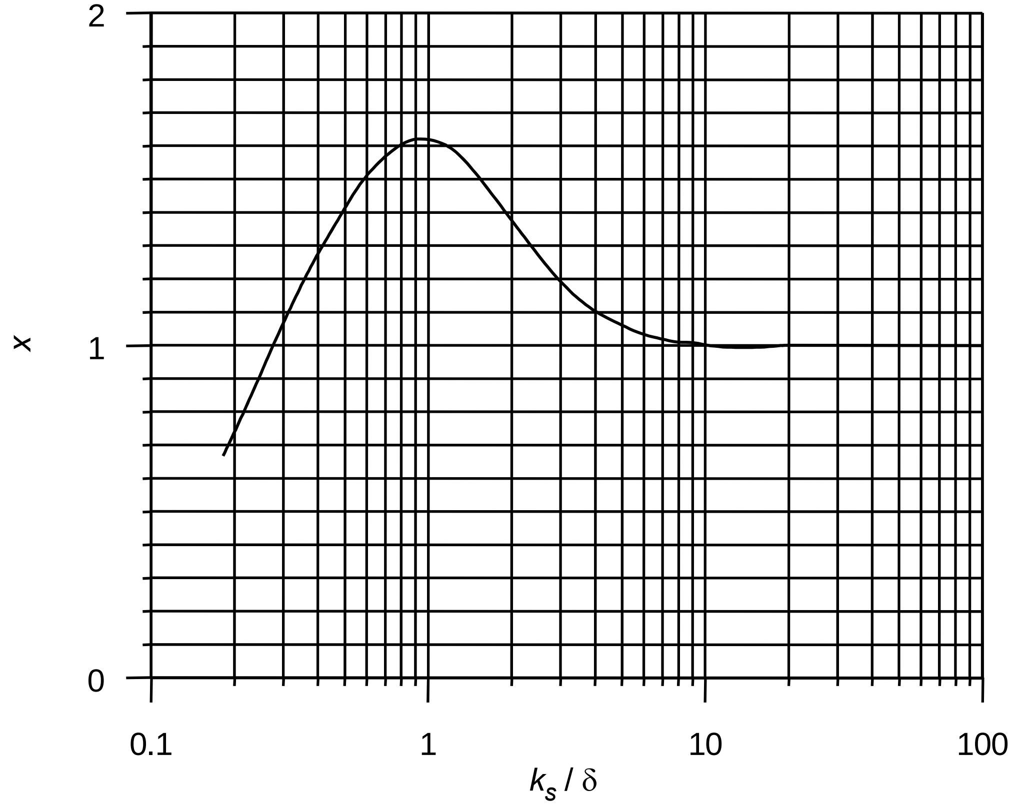

Fig. 3.3 Variation of correction factor x with k_s/δ, where k_s is the equivalent roughness height of Nikuradse (= d_{65})

- The lift force can be expressed as

$$F_L = 0.178 A_1 d^2 \frac{1}{2} \rho 5.75^2 g R_b' S \log^2 (10.6 X / \Delta_k)(1+\eta) \tag{3.23}$$

where R_b' = hydraulic radius due to grain roughness; such that shear velocity u_* =($g R_b' S$)$^{0.5}$

- The random function η represents the fluctuating component of the lift force being distributed according to the normal error law, where the standard deviation η_0 is a universal constant of η_0 = 0.5

$$\eta = \eta_0 \eta_* \tag{3.24}$$

where η_* = nondimensional number representing fluctuation of lift force

$$F_L = \frac{0.178 A_1 5.75^2}{2} \rho d^2 g R_b' S \log^2 (10.6 X / \Delta_k)(1+\eta_* \eta_0) \tag{3.25}$$

- The term probability p of erosion is expressed as the ratio of F_G to F_L, which has to be smaller than unity

$$1 > \frac{F_G}{F_L} = \left(\frac{1}{1+\eta_0 \eta_*}\right)\left(\frac{\Delta d}{R_b' S}\right)\left(\frac{2 A_2}{0.178 A_1 5.75^2}\right)\frac{1}{\log^2 (10.6 X / \Delta_k)} \tag{3.26}$$

- Using different symbols, Eq. (3.26) becomes

$$1 > \left(\frac{1}{1+\eta_0\eta_*}\right)\frac{\Psi B}{\beta_x^2} \tag{3.27}$$

where Ψ = flow intensity, that is $\Delta d/(R_b'S)$; $B = 2A_2/(0.178A_1 5.75^2)$; and $\beta_x = \log(10.6X/\Delta_k)$

- **Einstein** (1950) suggested two correction factors ξ and Y termed *hiding factor* and *lift correction factor*, respectively, being determined experimentally

- Small particles in sediment mass seem to hide between larger ones or in viscous sub-layer, such that their lift must be corrected by ξ^{-1}

- The hiding factor ξ of sediment particles is a function of d/X, where X is the characteristic distance (Fig. 3.4)

- The lift correction factor Y describes the change of lift coefficient in the sediment mass having different roughness and is a function of k_s/δ (Fig. 3.5)

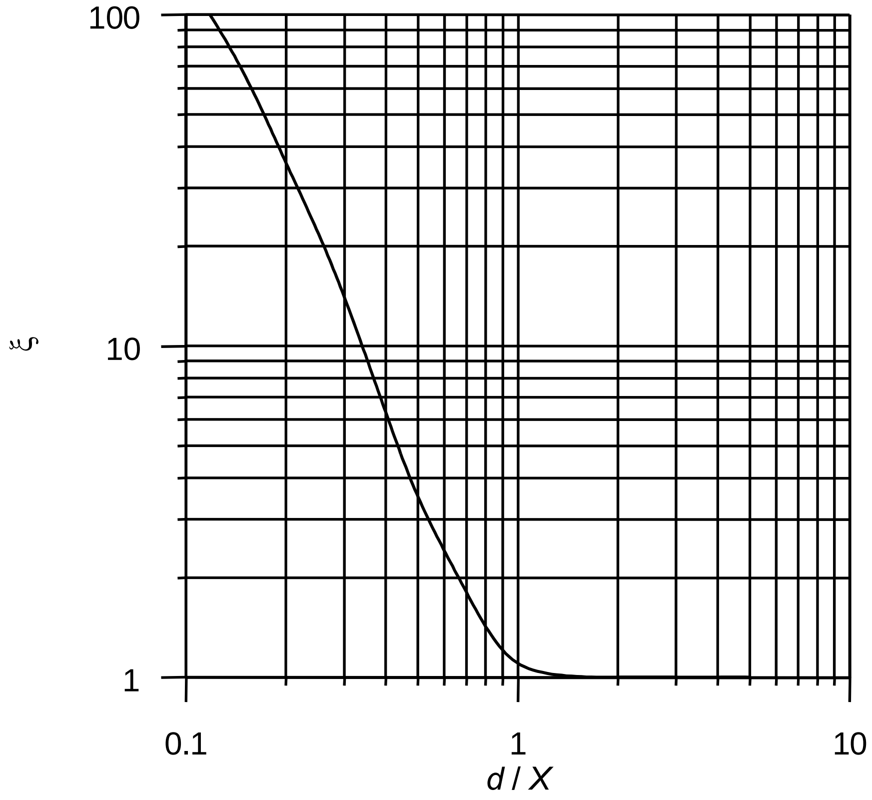

Fig. 3.4 Variation of hiding factor ξ with d/X

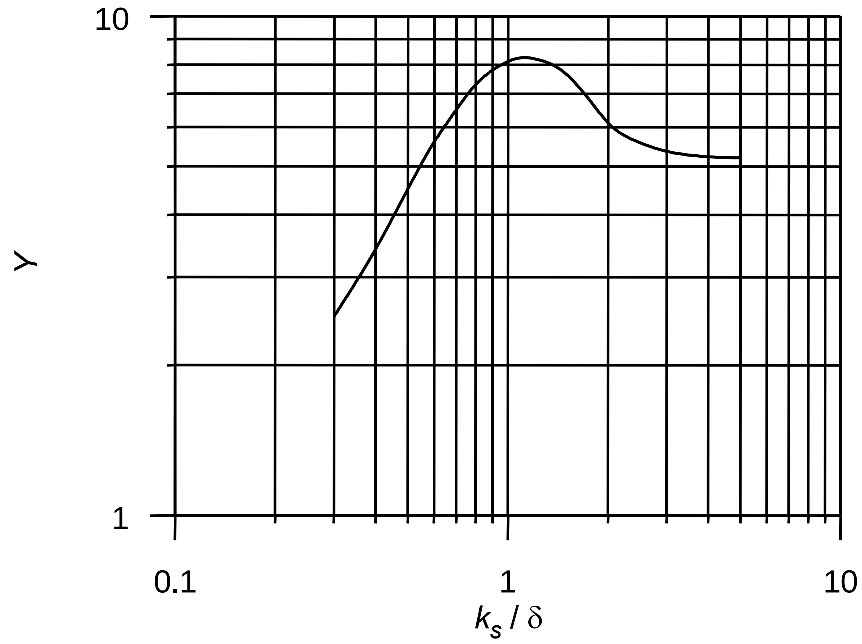

Fig. 3.5 Variation of lift correction factor Y with k_s/δ

- Whether the fluctuation of velocity is positive or negative, the lift force is always positive
- The inequality for the lift force can be modified as

$$\left|1+\eta_0\eta_*\right| > \xi Y B' \frac{\Psi\beta^2}{\beta_x^2} \tag{3.28}$$

where B' $=B/\beta^{2-}$; and β = log(10.6)

- Rearranging, it becomes

$$\left|\eta_* + \frac{1}{\eta_0}\right| > \frac{B'\Psi_*}{\eta_0} = B_*\Psi_* \tag{3.29}$$

where Ψ_* = $\xi Y\Psi(\beta/\beta_x)^2$; and B_* = B'/η_0

- The critical condition for particles to be removed from the bed is

$$\eta_* = \pm B_*\Psi_* - \frac{1}{\eta_0} \tag{3.30}$$

- Between the two values, no bed-load motion occurs
- Probability *p* of motion becomes

$$p = 1 - \frac{1}{\sqrt{\pi}} \int_{-B_*\Psi_* - \frac{1}{\eta_0}}^{B_*\Psi_* - \frac{1}{\eta_0}} \exp(-t^2)\,dt \tag{3.31}$$

- Equating Eqs. (3.20) and (3.31), the bed-load equation becomes

$$1 - \frac{1}{\sqrt{\pi}} \int_{-B_*\Psi_* - \frac{1}{\eta_0}}^{B_*\Psi_* - \frac{1}{\eta_0}} \exp(-t^2)\,dt = \frac{A_*\Phi}{1 + A_*\Phi} \tag{3.32}$$

- Experimentally determined $1/\eta_0 = 2$, $A_* = 43.5$ and $B_* = 1/7$

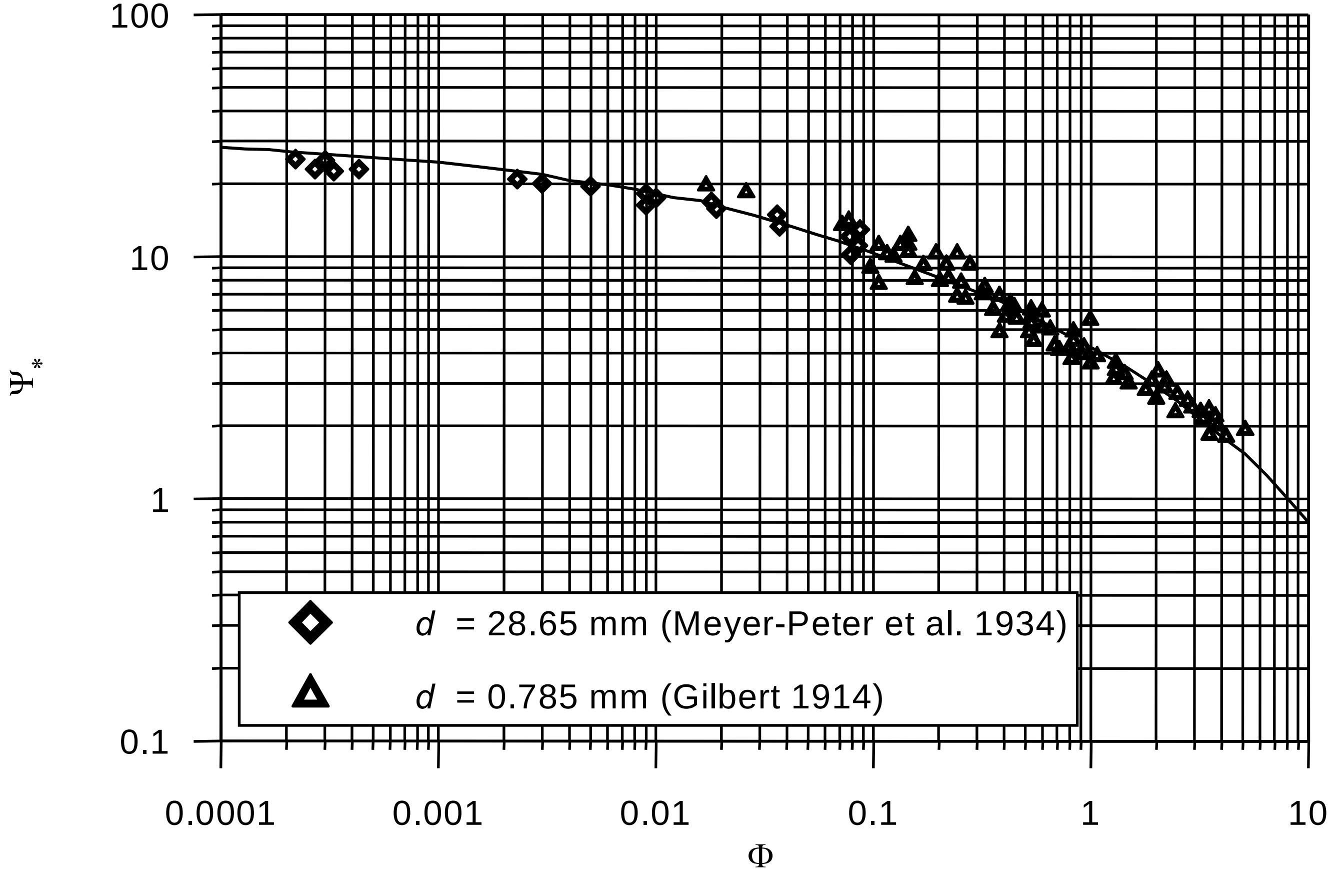

Fig. 3.6 Variation of Ψ_* with Φ obtained from **Einstein**'s (1950) Eq. (3.32)

Yalin's Bed-Load Equation

Yalin (1972) proposed a bed-load model from analysis of forces

- Let u_{bx} and u_{bz} represents velocity component of a sediment particle in the streamwise and normal directions
- The force components of flow acting on the particle in the streamwise direction F_x and normal direction F_z are

$$F_x = m' \frac{du_{bx}}{dt} \tag{3.33a}$$

$$-F_z - F_G = m' \frac{du_{bz}}{dt} \tag{3.33b}$$

where m' = submerged mass of the sediment particle

$$F_x = \frac{\pi}{8} C_{Dx} \rho d^2 (u - u_{bx})^2 \tag{3.34a}$$

$$F_z = \frac{\pi}{8} C_{Dz} \rho d^2 u_{bz}^2 \tag{3.34b}$$

where u = flow velocity received by the particle

- A particle jumps up from the bed under the action of a lift force F_L
- The lift force then decreases with distance from the bed and is equal to F_G at an elevation where the particle reaches its maximum vertical velocity component
- The maximum vertical velocity component can be obtained as

$$-F_z - F_G + F_L = m' \frac{du_{bz}}{dt} \qquad (3.35)$$

- To solve these equations, **Yalin** made the following assumptions:
  - $F_L/F_G \sim \exp(-z/d)$
  - C_{Dx} and C_{Dz} are constants
  - u/u_* is constant at the bed
- He obtained an expression for u_{bx}, and then he determined its average value over the time when it is in motion

$$\bar{u}_b = u_* C_1 \left[1 - \frac{1}{a_1 s_1} \ln(1 + a_1 s_1) \right] \qquad (3.36)$$

where $s_1 = (\Theta - \Theta_c)/\Theta_c$; $a_1 = 2.45\Theta_c^{0.5}/s^{0.4}$; and C_1 = a constant

- The parameter Θ, being the Shields parameter, is reciprocal of the parameter Ψ

- He determined the submerged weight of the bed-load per unit area W_s from the dimensional analysis

$$\frac{W_s}{(\rho_s - \rho)gd} = f_1(\Theta, R_*) \tag{3.37}$$

where $\Theta = \rho g R_b S/(\Delta \rho g d)$; R_b = hydraulic radius; $R_* = u_* d/\nu$; and ν = kinematic viscosity of fluid

- The particle Reynolds number can be expressed

$$R_* = \sqrt{\frac{\Delta g d^3}{\nu^2}\Theta} \tag{3.38}$$

Therefore, Eq. (3.37) can be rewritten

$$\frac{W_s}{(\rho_s - \rho)gd} = f_2\left(\Theta, \frac{\Delta g d^3}{\nu^2}\right) \tag{3.39}$$

- At the initiation of bed-load motion, $W_s = 0$

$$f_2\left(\Theta_c, \frac{\Delta g d^3}{\nu^2}\right) = 0 \qquad (3.40)$$

Combining Eqs. (3.39) and (3.40)

$$\frac{W_s}{(\rho_s - \rho) g d} = f_2(\Theta, \Theta_c) \qquad (3.41)$$

- **Yalin** assumed

$$\frac{W_s}{(\rho_s - \rho) g d} = C_2 s_1 \qquad (3.42)$$

where C_2 = constant to be determined

Substituting Eqs. (3.36) and (3.42) into Eqs. (3.33a) and (3.33b) and determining the constants from measured data, the bed-load transport rate g_b in weight per unit width is given by $g_b = g_{bs}(s/\Delta) = W_s\, \bar{u}_b\, (s/\Delta)$

- The bed-load equation of **Yalin** (1972) is

$$g_b = 0.635 \rho g s d u_* s_1 \left[1 - \frac{1}{a_1 s_1} \ln(1 + a_1 s_1) \right] \tag{3.43}$$

Bagnold's Approach

- **Bagnold** (1973) assumed: To sustain the saltation of a particle, the flowing fluid must act on the particle to provide a momentum component $m'u'$ with the time interval Δt between successive collisions of the particle with the bed, m' is the submerged mass and $-u'$ is the reduction in particle velocity in the direction of flow because of its collision with bed

- The flowing water has to exert a force on the particle with a component in the direction of flow

$$F_x = \frac{m'u'}{\Delta t} = \frac{F_G u'}{g\Delta t} \tag{3.44}$$

- If u_b is the average velocity of the particle, then the work done by the flowing fluid on the particle is $F_x u_b$

- Energy consumed in unit time by the flow is $F_G u_b \tan\varphi$, where φ is the frictional angle. Combining them

$$\frac{F_x}{F_G} = \tan\varphi = \frac{u'}{g\Delta t} \tag{3.45}$$

- If the flow velocity at z_n (at which the particle is acted upon by F_x) is u_n, then the difference of u_n and u_b is u_r (= $u_n - u_b$)
- If many particles move along the bed, then

$$Tu_b = F_G u_b \tan\varphi = g_{bs} \tan\varphi \tag{3.46}$$

where T = shear stress for maintaining sediment motion at $z = z_n$

So, the bed-load transport rate (in submerged weight) is

$$g_{bs} = \frac{T}{\tan\varphi}(u_n - u_r) \tag{3.47}$$

Using a coefficient a, the shear stress T is given by

$$T = a\tau_0 \tag{3.48}$$

- If the flow velocity follows the logarithmic law in the zone $z > z_n$, and the velocity at an elevation 0.4h from the bed is taken to be the average velocity, then

$$u_n = U - 5.75u_* \log\frac{0.4h}{z_n} \tag{3.49}$$

Using Eqs. (3.48) and (3.49) into Eq. (3.47), one gets

$$g_{bs} = \frac{a\tau_0}{\tan\varphi}\left[U - 5.75u_* \log\left(\frac{0.4h}{z_n}\right) - u_r\right] \tag{3.50}$$

Determination of *a*:

- Bagnold assumed *a* as follows

$$a = \frac{u_* - u_{*c}}{u_*} \tag{3.51}$$

where u_{*c} = critical shear velocity of the particle

Determination of u_r:

- The force exerted on a particle by the flow can be expressed

$$F_x = \frac{1}{2} C_{Dx} \frac{\pi}{4} d^2 \rho u_r^2 = F_G \tan\varphi \tag{3.52}$$

where C_{Dx} = drag coefficient

- For a particle falling in still water, a force F_z acts on the particle
- If the submerged weight of the particle is balanced by this force, the particle falls at a constant velocity w_{ss}

$$F_z = \frac{1}{2} C_{Dz} \frac{\pi}{4} d^2 \rho w_{ss}^2 = F_G \tag{3.53}$$

where C_{Dz} = drag coefficient for a settling particle

From Eqs. (3.52) and (3.53), one gets

$$u_r = w_{ss} (C_{Dz} \tan\varphi / C_{Dx})^{0.5} \tag{3.54}$$

- Measured data showed that $C_{Dz} \approx C_{Dx}$ and $\tan^{0.5}\varphi \approx 1$

Therefore, Eq. (3.54) becomes

$$u_r = w_{ss} \tag{3.55}$$

Determination of z_n:

- If no sand dunes form, the average elevation of the saltating particles is proportional to their diameter

$$z_n = m_1 d \tag{3.56}$$

where $m_1 = K_1(u_*/u_{*c})^{0.6}$ depending on the flow intensity

- In the laboratory, $K_1 = 0.4$ was found by **Francis** (1973). In rivers, it becomes 2.8 for sands and 7.3 - 9.1 for gravels (**Bagnold** 1977)

- Equation of bed-load obtained by **Bagnold** is

$$g_b = \frac{u_* - u_{*c}}{u_*} \cdot \frac{\tau_0 s U}{\Delta \tan\varphi}\left[1 - 5.75\left(\frac{u_*}{U}\right)\log\left(\frac{0.4h}{m_1 d}\right) - \left(\frac{w_{ss}}{U}\right)\right] \tag{3.57}$$

Note: $g_b = g_{bs}(s/\Delta)$.

Engelund and Fredsøe's Bed-Load Equation

Engelund and Fredsøe's (1976) model is applicable to the flow condition close to the threshold of sediment motion

- The bed-load particles are transported with a mean transport velocity $\bar{u}_b$
- The tractive or agitation force is given by

$$F_D = \frac{1}{2}\rho C_D \frac{\pi}{4} d^2 (\alpha u_* - \bar{u}_b)^2 \qquad (3.58)$$

where C_D = drag coefficient; and αu_* = flow velocity at particle level

- If the particle is at a distance of one to two particle diameters above the bed, α = 6 to 10
- The stabilizing frictional force on the moving particle is

$$F_s = \Delta\rho g \frac{\pi d^3}{6} \mu_d \qquad (3.59)$$

where μ_d = dynamic friction angle for the bed sediment

- For the equilibrium, the tractive force and the frictional force are equal

$$\frac{1}{2}\rho C_D \frac{\pi}{4} d^2 (\alpha u_* - \overline{u}_b)^2 = \Delta\rho g \frac{\pi d^3}{6} \mu_d \tag{3.60}$$

It gives

$$\frac{\overline{u}_b}{u_*} = \alpha\left[1 - \left(\frac{\Theta_0}{\Theta}\right)^{0.5}\right] \tag{3.61}$$

where $\Theta_0 = 4\mu_d/(3\alpha^2 C_D)$

- Θ_c is the critical value for the initial movement of a particle in a compactly arranged bed, and Θ_0 is the critical value for a particle protruding from the bed surface. Measured data showed $\Theta_0 = 0.5\Theta_c$

$$\frac{\overline{u}_b}{u_*} = \alpha\left[1 - 0.7\left(\frac{\Theta_c}{\Theta}\right)^{0.5}\right] \tag{3.62}$$

- **Engelund and Fredsøe** (1976) treated sediment particles as spheres of diameter d, so that there are approximately $1/d^2$ spherical particles in a unit area of bed surface

- For certain flow intensity, the portion of the particles on the bed surface that are moving is *p* (probability)
- Rate of bed-load transport is given by

$$g_b = \frac{\pi}{6} d^3 \rho_s g \frac{p}{d^2} \bar{u}_b \tag{3.63}$$

Using Eq. (3.62) into Eq. (3.63) yields

$$g_b = 10 \frac{\pi}{6} d^3 \rho_s g \frac{p}{d^2} \left[1 - 0.7 \left(\frac{\Theta_c}{\Theta} \right)^{0.5} \right] u_* \tag{3.64}$$

- According to **Bagnold**, the shear stress of flow is composed of particle shear stress τ and fluid shear stress τ'
- He suggested that the fluid shear stress τ' equals the critical bed shear stress for initiation of motion of bed particles

$$\tau = \tau_c + T = \tau_c + nF_x \tag{3.65}$$

where *n* = number of moving particles per unit area of bed surface; and F_x = drag force acting on the particles

- **Engelund** assumed

$$F_x = \frac{\pi d^3}{6} \Delta\rho g \mu_d \tag{3.66}$$

The results become

$$\Theta = \Theta_c + \frac{\pi}{6}\mu_d (nd^2) = \Theta_c + \frac{\pi}{6}\mu_d p \tag{3.67}$$

where $p = nd^2$

$$p = \frac{6}{\pi \mu_d}(\Theta - \Theta_c) \tag{3.68}$$

The bed-load equation is

$$g_b = 10 \frac{d}{\mu_d} \rho_s g \frac{u_*}{\Theta^{0.5}} (\Theta - \Theta_c)(\Theta^{0.5} - 0.7\Theta_c^{0.5}) \tag{3.69}$$

Transformation and Comparison of Bed-Load Equations

Meyer-Peter Equation:

- Eq. (3.13b) can be expressed according to **Chien** (1954) as

$$\Phi = 8\left(\frac{1}{\Psi} - 0.047\right)^{1.5} \qquad (3.70)$$

- For initiation of bed-load transport ($\Phi \to 0$), Θ_c = 0.047; and for a high bed-load transport ($\Theta >> \Theta_c$), $\Phi = 8/\psi^{1.5}$

Einstein Equation:

- Eq. (3.32) is written for $1/\eta_0 = 2$, $A_* = 43.5$ and $B_* = 1/7$ as

$$1 - \frac{1}{\sqrt{\pi}} \int_{-0.143\Psi-2}^{0.143\Psi-2} \exp(-t^2)dt = \frac{43.5\Phi}{1 + 43.5\Phi} \qquad (3.71)$$

Yalin Equation:

- Eq. (3.43) is transformed as

$$\Phi = 0.635 \frac{s_1}{\Psi}\left[1 - \frac{1}{a_1 s_1}\ln(1 + a_1 s_1)\right] \tag{3.72}$$

- For initiation of bed-load transport $\Theta \to \Theta_c$ (or very small) and $a_1 s_1$ is also small

$$\frac{1}{a_1 s_1}\ln(1 + a_1 s_1) \approx 1 - \frac{a_1 s_1}{2} \tag{3.73}$$

- The bed-load equation becomes

$$\Phi = 0.78 s^{0.4} \frac{\Psi_c^{1.5}}{\Psi^{0.5}}\left(\frac{1}{\Psi} - \frac{1}{\Psi_c}\right)^2 \tag{3.74}$$

- For a high intensity bed-load transport, Θ is large, and $a_1 s_1 \to \infty$

$$\ln(1 + a_1 s_1) \to 0 \tag{3.75}$$

- The bed-load equation becomes

$$\Phi = \frac{0.635}{\Psi^{1.5}}(\Psi_c - \Psi) \tag{3.76}$$

Bagnold Equation:

- Eq. (3.57) is transformed as

$$\Phi = \frac{1}{\Psi}\left(\frac{1}{\Psi^{0.5}} - \frac{1}{\Psi_c^{0.5}}\right)\left[\frac{1}{\tan\varphi}\left(5.75\log 30.2\frac{m_1 d}{h} - \frac{w_{ss}}{u_*}\right)\right] \tag{3.77}$$

Engelund and Fredsøe Equation:

- Assuming μ_d = 0.8 (for common river sands), Eq. (3.69) can be expressed as

$$\Phi = 11.6\left(\frac{1}{\Psi} - \frac{1}{\Psi_c}\right)\left(\frac{1}{\Psi^{0.5}} - \frac{0.7}{\Psi_c^{0.5}}\right) \tag{3.78}$$

- For a high bed-load transport $\Theta >> \Theta_c$, $\Phi = 11.6/\psi^{1.5}$

Comparative Results:

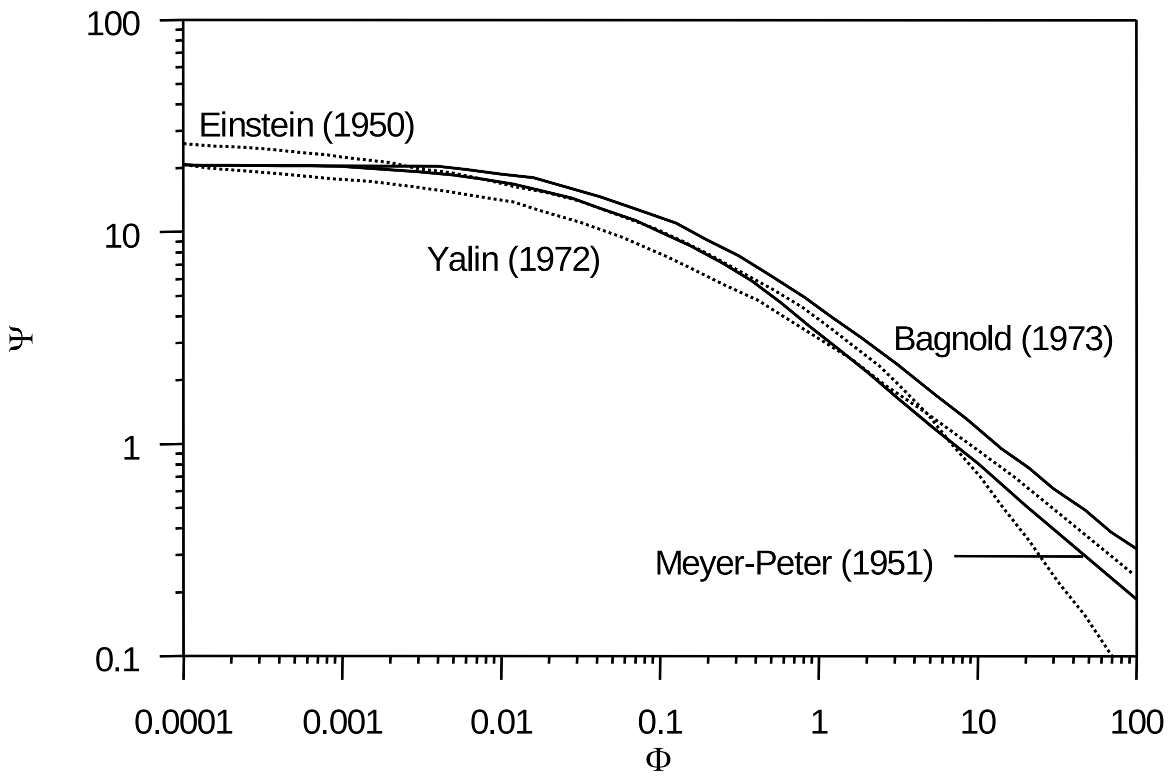

Fig. 3.7 Comparison of the equations of **Meyer-Peter**, **Einstein**, **Yalin** and **Bagnold**

- The Φ-ψ relationships for particle size of 0.2 mm and 2 mm are given and they give similar results
- The curve for **Bagnold** equation is the average of the two cases
- For $\psi > 2$, **Meyer-Peter**, **Einstein**, and **Bagnold** equations are close together, while **Yalin** equation yields smaller values for the bed-load transport
- **Engelund and Fredsøe** equation is good for bed-load near threshold condition

Characteristics of Particle Saltations

General characteristics of particle saltations after **Francis** (1973) and **Abbott and Francis** (1977):

- The saltation mode of transport is confined to a layer with a maximum thickness of about ten particle diameters, where the particle motion is dominated by the gravitational forces
- The particles receive their momentum directly from the flow pressure and viscous skin friction
- On the rising part of the trajectory, both the vertical component of the fluid drag force and the gravitational force are directed downwards
- During the falling part of the trajectory, the vertical component of the fluid drag force opposes the gravitational force
- The lift force is always directed upwards as long as the particle velocity lags behind the fluid velocity
- During impact of a particle with bed, most of its momentum is dissipated by particles of the bed in a sequence of horizontal impulses that may initiate rolling mode of transport known as *surface creep*

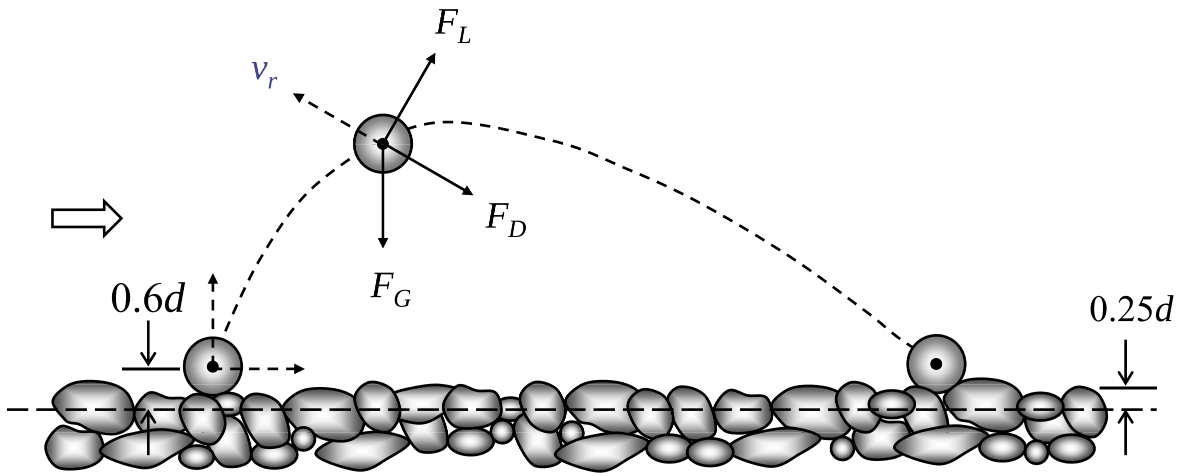

Fig. 3.8 Definition sketch of particle saltation

- The direction of the drag force F_D is opposed to the direction of the particle velocity v_r relative to the flow, while the lift component is in the normal direction

- Assuming the spherical particles and the forces due to fluid acceleration are of a second order (**Hinze** 1975), the equations of motion, according to **White and Schultz** (1977), can be written as

$$m_a \ddot{x} - F_L\left(\frac{\dot{z}}{v_r}\right) - F_D\left(\frac{u-\dot{x}}{v_r}\right) = 0 \tag{3.79a}$$

$$m_a \ddot{z} - F_L\left(\frac{u-\dot{x}}{v_r}\right) + F_D\left(\frac{\dot{z}}{v_r}\right) + F_G = 0 \quad (3.79b)$$

where m_a = particle mass and added fluid mass; v_r = particle velocity relative to the flow, that is $[(u - \dot{x})^2 + \dot{z}^2]$; u = local flow velocity; $\dot{x}$ and $\dot{z}$ = streamwise and vertical particle velocities, respectively; and $\ddot{x}$ and $\ddot{z}$ = streamwise and vertical particle acclerations, respectively

- The total mass of the spherical particle can be represented

$$m_a = \frac{1}{6}(\rho_s + \alpha_m \rho)\pi d^3 \quad (3.80)$$

where α_m = added mass coefficient

- Assuming potential flow, the added mass α_m of a perfect sphere is exactly equal to the half the mass of the fluid displaced by the sphere

- When the flow is separated from the solid sphere, α_m may be different. Here, α_m may be considered as 0.5

- The drag force F_D, which is caused by pressure and viscous skin friction, can be expressed as

$$F_D = \frac{1}{2} C_D \frac{\pi d^2}{4} \rho v_r^2 \quad (3.81)$$

- The drag coefficient C_D can be determined from the empirical expressions given by **Morsi and Alexander** (1972)

- The lift force in a shear flow is caused by the velocity gradient present in the flow (shear flow) and by the spinning motion of the particle (Magnus effect)

- For a sphere moving in a viscous flow, **Saffman** (1968) derived the lift due to shear as

$$F_L\,(\text{shear lift}) = C_L \rho \nu^{0.5} d^2 v_r \left(\frac{\partial u}{\partial z}\right)^{0.5} \quad (3.82)$$

- The lift force due to spinning motion in a viscous flow determined by **Rubinow and Keller** (1961) is given by

$$F_L\,(\text{Magnus lift}) = C_L \rho d^3 v_r \omega \quad (3.83)$$

where ω = angular velocity of the particle

- The submerged weight of the particle is

$$F_G = \frac{\pi}{6} d^3 \Delta\rho g \qquad (3.84)$$

- The flow velocity distribution assumed to follow logarithmic law is given by

$$u = \frac{u_*}{\kappa} \ln\left(\frac{z}{z_0}\right) \qquad (3.85)$$

where κ = von Karman constant (= 0.4); z_0 = zero-velocity level, that is $0.11(\nu/u_*) + 0.03k_s$

Boundary Conditions and Solution Scheme:

- The bed level is assumed at a distance of 0.25d below the top level of the bed particles
- The initial position of the particle is 0.6d above the bed level
- Experiments of **Francis** (1973) and **Abbott and Francis** (1977) showed $\dot{x} = \dot{z} = 2u_*$
- Eqs. (3.79a) and (3.79b) can be transformed to a system of ordinary simultaneous differential equations of the first order
- The system can be solved numerically by means of an automatic step-change differential equation solver

SUSPENDED-LOAD TRANSPORT

- *Suspended-load* refers to sediment particles that are supported by the upward component of turbulent flow and stay in suspension for an appreciable period of time
- The suspended-load transport rate can be determined as

$$q_s = \int_a^h cu\,dz \qquad (4.1a)$$

$$g_s = \rho_s g \int_a^h cu\,dz \qquad (4.1b)$$

where q_s = suspended-load transport in volume per unit time and width; g_s = suspended-load transport in weight per unit time and width; u = time-averaged velocity at an elevation z; c = concentration by volume at an elevation z; a = thickness of bed-load transport; h = flow depth; ρ_s = mass density of sediment; and g = gravitational acceleration

Diffusion Theory of Suspension

- The solutions developed for molecular diffusion are by analogy important for turbulent diffusion
- Analysis of molecular diffusion is based on the continuum hypothesis and Fick's law

$$P = -\varepsilon_m \frac{\partial C}{\partial z} \qquad (4.2)$$

where P = rate at which the quantity is transported across unit area normal to z-direction; ε_m = diffusion coefficient; and C = concentration of the quantity transported by diffusion

- Introducing the requirement of the conservation of matter, Eq. (4.2) becomes

$$\frac{\partial C}{\partial t} = -\frac{\partial P}{\partial z} = \varepsilon_m \frac{\partial^2 C}{\partial z^2} \qquad (4.3)$$

where t = time

Eq. (4.3) has a solution

$$C(z,t) = \frac{B}{t^{0.5}} \exp\left(-\frac{z^2}{4\varepsilon_m t} \right) \tag{4.4}$$

where B = integration constant

- In presence of flow, the Fick's law is generalized to $\partial C/\partial t + \nabla \cdot (Cu) = \varepsilon_m \nabla^2 C$, and then for incompressible flow, it becomes $\partial C/\partial t + u \cdot \nabla C = \varepsilon_m \nabla^2 C$ or

$$\frac{\partial C}{\partial t} + u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} + w\frac{\partial C}{\partial z} = \varepsilon_m \frac{\partial^2 C}{\partial x^2} + \varepsilon_m \frac{\partial^2 C}{\partial y^2} + \varepsilon_m \frac{\partial^2 C}{\partial z^2} \tag{4.5}$$

In tensor form, Eq. (4.5) becomes

$$\frac{\partial C}{\partial t} = -u_i \frac{\partial C}{\partial x_i} + \varepsilon_m \frac{\partial^2 C}{\partial x_i \partial x_i} \tag{4.6}$$

where x_i = rectangular coordinate system for i = 1, 2 and 3. Here, ε_m refers to molecular diffusion. For dispersion in a turbulent flow field, $C = \overline{C} + C'$ and $u_i = \overline{u}_i + u_i'$, where $\overline{C}$ and $\overline{u}_i$ = time-averaged concentration and velocity at a given point; and C' and u_i' = fluctuations of C and u_i, respectively

- Substituting C and u_i in Eq. (4.6) and using Reynolds conditions, one obtains

$$\frac{\partial \overline{C}}{\partial t} = -\overline{u}_i \frac{\partial \overline{C}}{\partial x_i} - \frac{\partial}{\partial x_i}(\overline{C' u_i'}) + \varepsilon_m \frac{\partial^2 \overline{C}}{\partial x_i \partial x_i} \qquad (4.7)$$

- **Elder** (1959) found it convenient to define a coefficient of turbulent diffusion such that

$$(\varepsilon_t)_{ij} \frac{\partial \overline{C}}{\partial x_j} = -\overline{C' u_i'} \qquad (4.8)$$

- Under the assumption that molecular and turbulent diffusions are independent and thus additive

$$\varepsilon_{ij}(x_i) = (\varepsilon_t)_{ij} + \varepsilon_m \qquad (4.9)$$

- In open channel flow, the turbulent diffusivity is usually considerably larger than the molecular one
- Time-averaged value is no longer required and therefore dropped
- The scalar ε_i replaces ε_{ij} that refers to as the diffusion tensor

$$\frac{\partial C}{\partial t} = -u_i \frac{\partial C}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\varepsilon_i \frac{\partial C}{\partial x_i}\right) \qquad (4.10)$$

Vertical Distribution of Suspended Particles

- The concept of an analogy between the process of mass and momentum transfer in a turbulent flow is known as the *Reynolds analogy*
- Considering the transfer of momentum and mass in x_3-direction

$$\text{Momentum flux} = \rho(\varepsilon_M + \nu)\frac{\partial u_3}{\partial x_3} = \rho\varepsilon_M \frac{\partial u_3}{\partial x_3} \quad (4.11a)$$

$$\text{Mass flux} = (\varepsilon_t + \varepsilon_m)\frac{\partial C}{\partial x_3} = \varepsilon_3 \frac{\partial C}{\partial x_3} \quad (4.11b)$$

where ρ = mass density of fluid; and ν = kinematic viscosity of fluid

- Under the assumption that $\varepsilon_M > \nu$ and $\varepsilon_m > \varepsilon_t$, the Reynolds analogy is valid if the mechanisms which control both the mass and momentum transfers are in fact identical

- As this is most likely the case, one can use ε_M and ε_3 interchangeable in the x_3-direction

$$\varepsilon_M = \varepsilon_3 \qquad (4.12)$$

- If and only if the solid particles follow the motion of the fluid particles, equality between the diffusivity of fluid mass ε_3 and the diffusivity of suspended solid mass ε_{s3} exists

$$\varepsilon_{s3} = \beta\varepsilon_3 \qquad (4.13)$$

where β = factor of proportionality

- Experimental data revealed that β is unity

Uniform Turbulence Distribution at Steady-State Condition

- For steady condition $\partial C/\partial t = 0$
- Assuming sediment concentration (by weight) $C = C(z)$ and ε_s (ε_{s3} replaced by ε_s) being independent of z, Eq. (4.10) can be expressed as

$$0 = Cw_{ss} + \varepsilon_s \frac{dC}{dz} \qquad (4.14)$$

where w_{ss} = settling velocity of the sediment particles

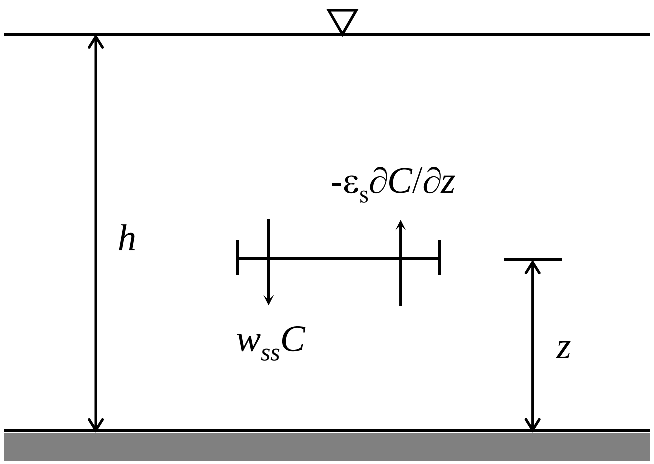

Fig. 4.1 Settling and diffusion of sediment

- The solution of Eq. (4.14) is

$$\frac{C}{C_a} = \exp\left[-\frac{w_{ss}(z-a)}{\varepsilon_s}\right] \quad (4.15)$$

where C_a = a reference concentration (by weight) at a distance *a* from the bed

- For high concentration, Eq. (4.14) must be modified to take into account the sediment particles occupy a certain fraction of the total volume

- A certain volume of sediment $w_{ss}C$ settles through a unit area, this volume is replaced from below by the fluid and sediment. The concentration is also approximately *C*, so the volume of sediment transported up through the unit area is $C(w_{ss}C)$

$$0 = C(1-C)w_{ss} + \varepsilon_s \frac{dC}{dz} \quad (4.16)$$

Nonuniform Turbulence Distribution at Steady-State Condition

- Separating the variables, Eq. (4.14) can be rearranged as

$$\frac{dC}{C} + w_{ss}\frac{dz}{\varepsilon_s} = 0 \qquad (4.17)$$

- The diffusivity of solid particles ε_s is given as a function of *z*, that is $\varepsilon_s = \varepsilon_s(z)$. Integrating Eq. (4.17) yields

$$C = C_a \exp\left(-w_{ss}\int_a^z \frac{dz}{\varepsilon_s}\right) \qquad (4.18)$$

- For turbulent flow, the Reynolds stress τ can be expressed as

$$\tau = \varepsilon\rho\frac{du}{dz} \qquad (4.19)$$

where ε = eddy viscosity or momentum diffusion coefficient of fluid

- The Reynolds stress distribution along *z* is given by

$$\tau = \tau_0\left(1 - \frac{z}{h}\right) \qquad (4.20)$$

where τ_0 = bed shear stress

- Assuming that logarithmic velocity distribution is valid

$$\frac{du}{dz} = \frac{u_*}{\kappa z} \tag{4.21}$$

where u_* = shear velocity; and κ = von Karman constant (= 0.4)

From Eq. (4.19) – (4.21), one gets

$$\varepsilon_z = \kappa u_* (h - z)\frac{z}{h} \tag{4.22}$$

Eq. (4.13) suggests that

$$\varepsilon_s = \beta \kappa u_* (h - z)\frac{z}{h} \tag{4.23}$$

- Inserting ε_s from Eq. (4.23) to Eq. (4.18) and integrating

$$\frac{C}{C_a} = \left(\frac{h - z}{z} \cdot \frac{a}{h - a}\right)^{\zeta} \tag{4.24}$$

where $\zeta = w_{ss}/(\kappa u_*)$

- The concentration distribution equation was introduced by **Rouse** (1937)
- It can be used to calculate the concentration of a given w_{ss} of the sediment size at any distance z from the bed if a reference concentration C_a at a distance a is known
- The suspended-load of sediment is given by

$$g_s = \int_a^h C u dz \qquad (4.25)$$

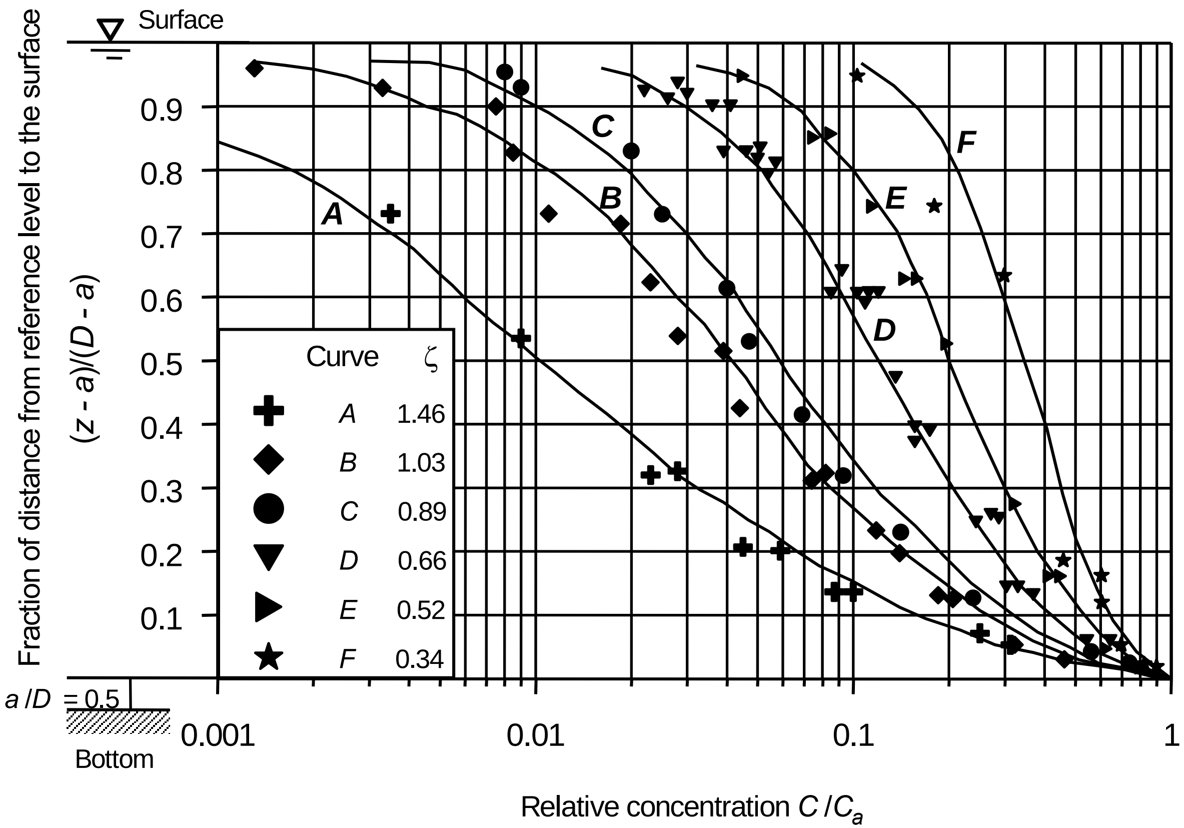

Fig. 4.2 Vertical distribution of suspended sediment concentration

- At the bed (z = 0), the concentration C becomes infinity breaking down the Eq. (4.24)
- **Einstein et al.** (1940) suggested that the suspension is not possible in the so-called *bed-layer*, which has a thickness of 2d
- For low values of ζ, the sediment distribution is nearly uniform, while for large values of ζ, little sediment is found at the free surface

Estimation of C_a:

- The depth a and concentration C_a in Eq. (4.24) are called as *reference elevation* and *reference sediment concentration*
- The reference elevation a is the boundary between the bed load and the suspended load
- **Bijker** (1992) suggests that a is taken as the bed roughness k_s and relates C_a to the bed-load transport q_b
- It is assumed that bed-load transport takes place in the bed-load layer from $z = 0$ to $z = a = k_s$, and in this layer, there is a constant sediment concentration C_a
- He argues that in hydraulically rough flow there is still a viscous sub-layer, which starts from $z = 0$ to $z = z_\delta$ where the linear velocity distribution is tangent with the logarithmic velocity distribution
- He estimated the depth-averaged velocity $\bar{u}_a$ up to depth $z = a$ (= k_s), as $\bar{u}_a \approx 6.34u_*$. Given bed-load $q_b = C_a \bar{u}_a k_s$, the sediment concentration C_a is estimated as $C_a = q_b/(6.34u_* k_s)$

Sediment Concentration at Free Surface

- In Eq. (4.24), the sediment concentration *C* at the free surface $z = h$ is zero
- ε_z is zero at free surface, but ε_s is finite there
- For momentum exchange, the relationship of the Reynolds stress $\tau = \rho\,\overline{u'v'}$ holds
- Sediment suspension depends primarily on v', which is much less than u'
- At the free surface logarithmic law of velocity distribution does not hold
- Following equation makes possible to estimate the velocity near free surface

$$\frac{u_{max} - u}{u_*} = \frac{2}{\kappa} \operatorname{arctanh}\left(\frac{h - z}{h}\right)^{1.5} \qquad (4.26)$$

where u_{max} = maximum value of u which occurs at $z = h$

- The mixing length *l* and momentum exchange coefficient ε_z are

$$l = \frac{\kappa}{3} h \left[1 - \left(\frac{h-z}{h} \right)^3 \right] \tag{4.27}$$

$$\varepsilon_z = \frac{\kappa}{3} u_* h \sqrt{\frac{h-z}{h}} \left[1 - \left(\frac{h-z}{h} \right)^3 \right] \tag{4.28}$$

- Using the relationship $\varepsilon_s = \beta\varepsilon_z$, the differential equation is

$$C w_{ss} + \beta \frac{\kappa}{3} u_* h \sqrt{\frac{h-z}{h}} \left[1 - \left(\frac{h-z}{h} \right)^3 \right] \frac{dC}{dz} = 0 \tag{4.29}$$

The solution of Eq. (4.29) is

$$\frac{C}{C_a} = \exp(\zeta_\beta \Omega) \tag{4.30}$$

where $\zeta_\beta = w_{ss}/(\beta\kappa u_*)$

$$\Omega = 0.5 \ln \frac{\left[\left(\frac{h-z}{h} \right)^{0.75} + 1 \right] \left[\left(\frac{h-z}{h} \right)^{0.5} - 1 \right]^3}{\left[\left(\frac{h-z}{h} \right)^{1.5} - 1 \right] \left[\left(\frac{h-z}{h} \right)^{0.5} + 1 \right]^3} + \sqrt{3} \arctan \left[-\frac{h}{z} \sqrt{\frac{3(h-z)}{h}} \right]_{z=a}^{z} \tag{4.31}$$

Influence of Sediment Suspension on Velocity and Resulting Concentration

Velocity Distribution

- **Einstein and Chien** (1955) modified the traditional logarithmic law of the velocity distribution due to the influence of sediment suspension
- The zone close to the bed, where the sediment concentration is high, referred to as *heavy-fluid zone*
- The remaining portion of the flow, where the sediment concentration is relative low, has no change of fluid mass density and is called as *light-fluid zone*
- The clear water flow follows the logarithmic law of velocity distribution

$$\frac{u}{u_*} = \frac{2.3}{\kappa} \ln\left(30.2 \frac{z}{k_s}\right) \qquad (4.32)$$

where κ = von Karmans constant; and k_s = equivalent sand roughness of Nikuradse

- Eq. (4.32) was derived assuming that the Reynolds stress is

$$\tau = \rho \varepsilon_z \frac{du}{dz} \qquad (4.33)$$

- In sediment-laden flow, a more reasonable velocity distribution can be obtained by the inclusion of the participation of solid particles in the exchange mechanism

- **Einstein et al.** (1955) derived the following relationship

$$\tau = \left(1 + \frac{\rho_s - \rho}{\rho} C\right) \rho \varepsilon_z \frac{du}{dz} \qquad (4.34)$$

- Within the light-fluid zone of the small concentration, Eq. (4.34) becomes Eq. (4.33)

- Under these circumstances an equation similar to the clear water equation Eq. (4.32), but with different numerical constants

- Experiments suggested the following relationship

$$\frac{u}{u_*} = 17.66 + \frac{2.3}{\kappa} \ln\left(\frac{z}{35.45 k_s}\right) \qquad (4.35)$$

- Experiments revealed that close to the bed, whenever the local sediment concentration reaches a value of 981 N/m^3 or $z/h < 0.1$, Eq. (4.35) fails
- Shear stress given by Eq. (4.34) can be approximated by τ_0 as

$$\tau_0 = \int_0^h [\rho + (\rho_s - \rho)C]\, gS\, dz \tag{4.36}$$

where S = energy slope

- The velocity distribution is thus obtained as

$$\frac{u}{u_*} = \frac{2.3}{\kappa} \cdot \frac{\sqrt{1 + \dfrac{\rho_s - \rho}{\rho} \cdot \dfrac{1}{h} \int_0^h C dz}}{\sqrt{1 + \dfrac{\rho_s - \rho}{\rho} C_a}} \ln\left(A_e \frac{z}{k_s} \right) \tag{4.37}$$

where C_a = sediment concentration at the surface of the bed layer; and A_e = constant to be determined

- The depth averaged velocity U can be obtained from Eq. (4.35)

$$\frac{U}{u_*} = 17.66 + \frac{2.3}{\kappa} \ln\left(\frac{h}{96.5 k_s} \right) \tag{4.38}$$

Sediment Distribution

- Without lacking of generality, Eq. (4.5) can be written as

$$\frac{\partial C}{\partial t} = -u_i \frac{\partial C}{\partial x_i} - C\frac{\partial u_i}{\partial x_i} + \frac{\partial}{\partial x_i}\left(\varepsilon_i \frac{\partial C}{\partial x_i}\right) \tag{4.39}$$

- For the special case of uniform flow in x_1-direction and the concentration being constant with time, the variation in $x_3 = z$ component are considered for which $u_3 = w$

$$0 = -w\frac{\partial C}{\partial z} - C\frac{\partial w}{\partial z} + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial C}{\partial z}\right) \tag{4.40}$$

- The rate of change of suspended matter is given by

$$0 = -w_s \frac{\partial C}{\partial z} - C\frac{\partial w_s}{\partial z} + \frac{\partial}{\partial z}\left(\varepsilon_s \frac{\partial C}{\partial z}\right) \tag{4.41}$$

where w_s = velocity of solid particle in z-direction. For the fluid by

$$0 = -w\frac{\partial C}{\partial z} - (1-C)\frac{\partial w}{\partial z} + \frac{\partial}{\partial z}\left(\varepsilon_z \frac{\partial C}{\partial z}\right) \tag{4.42}$$

- The velocity relationship can be given by

$$w_s = w - w_{ss} \tag{4.43}$$

Eliminating w_s and w from Eqs. (4.41) and (4.42)

$$[\varepsilon_s + C(\varepsilon_z - \varepsilon_s)]\frac{\partial C}{\partial z} + (1-C)Cw_{ss} = 0 \qquad (4.44)$$

where ε_s and ε_z = diffusivity of solid matter and liquid matter

- To simplify the solution, the diffusion coefficients of solid and liquid matter are assumed same, that is $\varepsilon_s = \varepsilon_z$

$$\varepsilon_s \frac{dC}{dz} + (1-C)Cw_{ss} = 0 \qquad (4.45)$$

The solution of Eq. (4.45) is

$$\left(\frac{C}{1-C}\right)\left(\frac{1-C_a}{C_a}\right) = \left(\sqrt{\frac{1-z/h}{1-a/h}} \cdot \frac{B_s - \sqrt{1-a/h}}{B_s - \sqrt{1-y/h}}\right)^{\zeta_0} \qquad (4.46)$$

where $\zeta_0 = w_{ss}/(\kappa_s B_s u_*)$; B_s = constant of integration in the velocity distribution law ($B_s \leq 1$); and κ_s = constant similar to von Karman constant

- For large sediment concentration, Eq. (4.45) should be used as

$$\frac{dC}{dz} + \left(1 + \frac{\rho_s - \rho}{\rho} C\right)(1 - C) C w_{ss} \frac{\rho}{\tau} \cdot \frac{du}{dz} = 0 \qquad (4.47)$$

- For small sediment concentration, as encountered in the light-fluid zone, Eq. (4.45) reduces to

$$\varepsilon_s \frac{dC}{dz} + C w_{ss} = 0 \qquad (4.48)$$

Suspended-Load by Diffusion Theory

Lane and Kalinske's Approach

- **Lane and Kalinske** (1941) assumed $\varepsilon_s = \varepsilon_z$ and $\beta = 1$, Eq. (4.23) becomes

$$\varepsilon_s = \kappa u_* (h - z)\frac{z}{h} \tag{4.49}$$

- The average value of ε_s along z is

$$\bar{\varepsilon}_s = \frac{1}{h}\int_0^h \varepsilon_s dz = \frac{\kappa u_*}{h^2}\int_0^h (h - z) z dz \tag{4.50}$$

- Integrating Eq. (4.50) and using the von Karman constant $\kappa = 0.41$

$$\bar{\varepsilon}_s = \frac{1}{15} u_* h \tag{4.51}$$

- Introducing Eq. (4.51) into Eq. (4.15)

$$\frac{C}{C_a} = \exp\left[-\frac{15 w_{ss}}{u_*}\left(\frac{z - a}{h}\right)\right] \tag{4.52}$$

- The suspended-load (by weight) per unit time and width is given by

$$g_s = \int_0^h Cu\,dz \qquad (4.53)$$

Using Eq. (4.52) into Eq. (4.53)

$$g_s = qC_a P_L \exp\left(\frac{15 w_{ss} a}{u_* h}\right) \qquad (4.54)$$

where q = flow discharge per unit width; and P_L = function of w_{ss}/u_* and relative roughness $n/h^{1/6}$, where n = Manning roughness coefficient

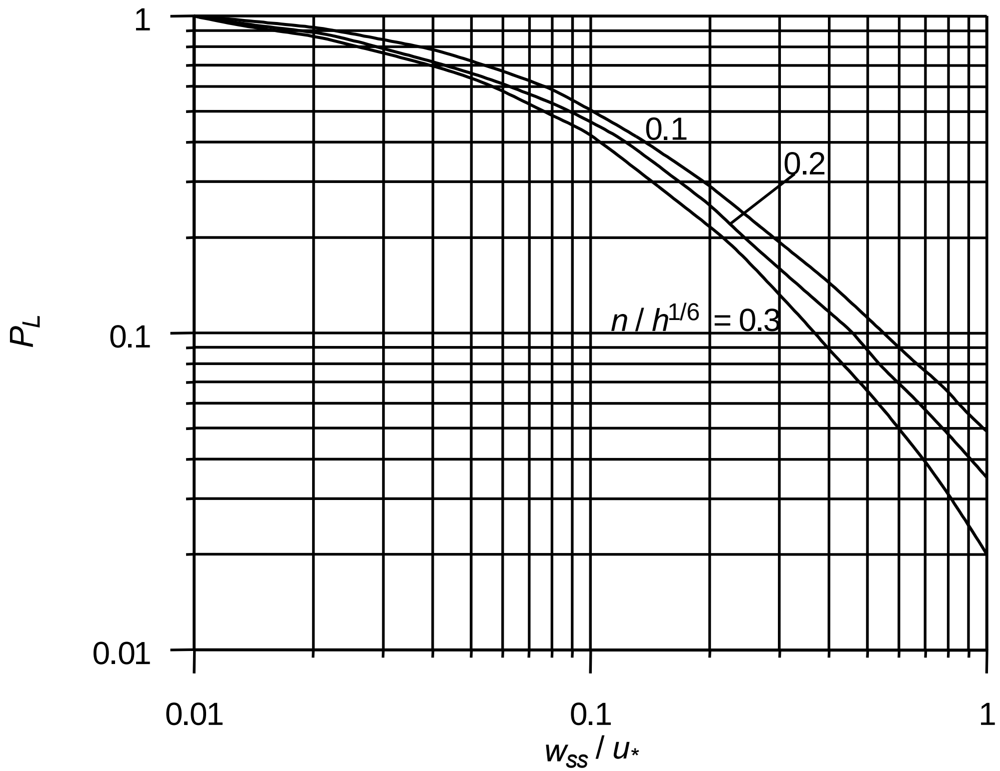

Fig. 4.3 Relationship of P_L after **Lane and Kalinske** (1941)

Einstein's Approach

- **Einstein** (1950) assumed that $\beta = 1$ and $\kappa = 0.4$
- Replacing shear velocity u_* with shear velocity due to grain roughness u'_*

$$\zeta_\beta = \zeta = \frac{w_{ss}}{0.4u'_*} \tag{4.55}$$

- The velocity can be expressed

$$\frac{u}{u'_*} = 5.75\log\left(30.2\frac{z}{\Delta_k}\right) \tag{4.56}$$

where $\Delta_k = k_s/x = d_{65}/x$; and x = correction factor

Substituting Eqs. (4.24) and (4.56) into Eq. (4.25)

$$g_s = \int_a^h C_a\left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^\zeta 5.75u'_*\log\left(30.2\frac{z}{\Delta_k}\right)dz \tag{4.57}$$

Replacing a with $E = a/h$ and z with $z' = z/h$

$$g_s = \int_E^1 Cuhdz' = hu'_*C_a\left(\frac{E}{1-E}\right)^\zeta 5.75\int_E^1\left(\frac{1-z}{z}\right)^\zeta \log\left(\frac{30.2z}{\Delta_k/h}\right)dz \tag{4.58}$$

Eq. (4.58) becomes

$$g_s = 5.75 C_a u'_* h \left(\frac{E}{1-E}\right)^{\zeta} \left[\log\left(\frac{30.2z}{\Delta_k}\right) \int_E^1 \left(\frac{1-z}{z}\right)^{\zeta} dz + 0.434 \int_E^1 \left(\frac{1-z}{z}\right)^{\zeta} \ln z \, dz \right] \quad (4.59)$$

- As the closed-form integration of Eq. (4.59) is impossible, **Einstein** (1950) expressed it as

$$g_s = 11.6 C_a u'_* a \left[\log\left(\frac{30.2h}{\Delta_k}\right) I_1 + I_2 \right] \quad (4.60)$$

$$I_1 = 0.216 \frac{E^{\zeta-1}}{(1-E)^{\zeta}} \int_E^1 \left(\frac{1-z}{z}\right)^{\zeta} dz \quad (4.61a)$$

$$I_2 = 0.216 \frac{E^{\zeta-1}}{(1-E)^{\zeta}} \int_E^1 \left(\frac{1-z}{z}\right)^{\zeta} \ln z \, dz \quad (4.61b)$$

- Bed-load rate of a given size i_b is $i_b g_b$
- If the velocity with which the bed-load moves is u_b, then the weight of particles of a given grain size per unit area is $i_b g_b / u_b$
- Average concentration in the layer is given by

$$C_a = A_s \frac{i_b g_b}{a u_b} \quad (4.62)$$

- The average bed-load velocity u_b was assumed to be proportional to shear velocity due to grain roughness u'_*
- Eq. (4.62) becomes

$$C_a = \frac{1}{11.6} \cdot \frac{i_b g_b}{a u'_*} \tag{4.63}$$

- The suspended-load equation for each fraction, where a bed-load function exists

$$i_s g_s = i_b g_b \left[\log\left(\frac{30.2h}{\Delta_k}\right) I_1 + I_2 \right] = i_b g_b (P_E I_1 + I_2) \tag{4.64}$$

where i_s = size fraction in suspension; and P_E = 2.303log(30.2h/Δ_k), transport parameter

Brook's Approach

- **Brooks** (1963) assumed that the logarithmic velocity distribution is applicable and sediment concentration follows Eq. (4.24)

$$g_s = C_{0.5h} q \left[\left(1 + \frac{u_*}{\kappa U} \right) \int_E^1 \left(\frac{1-z}{z} \right)^{\zeta_\beta} dz + \frac{u_*}{\kappa U} \int_E^1 \left(\frac{1-z}{z} \right)^{\zeta_\beta} \ln z \, dz \right] \qquad (4.65)$$

where q = flow discharge per unit width; and $C_{0.5h}$ = reference sediment concentration at $y = 0.5h$

- Eq. (4.65) can be expressed in terms of a transport function T_B

$$\frac{g_s}{q C_{0.5h}} = T_B \left(\frac{\kappa U}{u_*}, \zeta_\beta, E \right) \qquad (4.66)$$

- Taking a lower limit of integration at $u = 0$, E becomes

$$E = \exp\left(-\frac{\kappa U}{u_*} - 1 \right) \qquad (4.67)$$

- Eq. (4.66) reduces to

$$\frac{g_s}{qC_{0.5h}} = T_B\left(\frac{\kappa U}{u_*}, \zeta_\beta\right) \qquad (4.68)$$

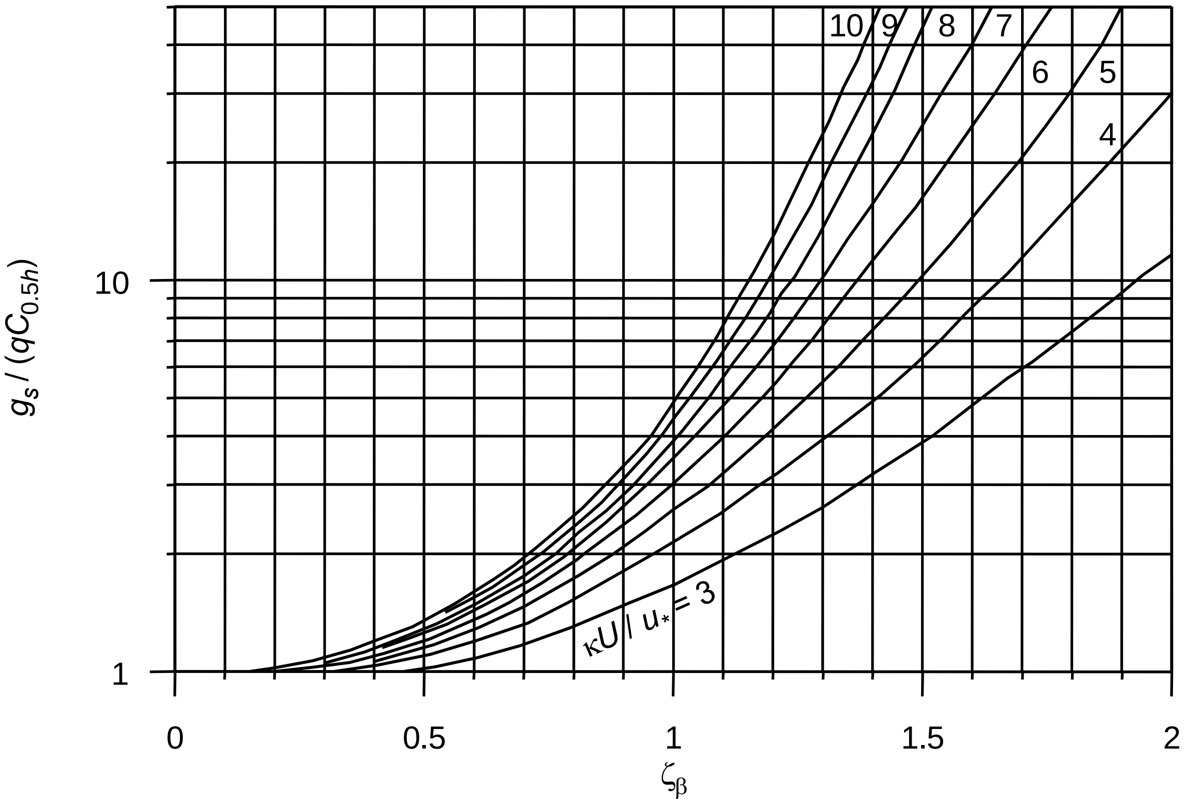

Fig. 4.4 **Brook**'s (1963) suspended-load transport function

Chang et al.'s Approach

- **Chang et al.** (1965) assumed that Eq. (4.23) holds good and rewrote as

$$\varepsilon_s = \beta \kappa u_* h \xi (1-\xi)^{0.5} \tag{4.69}$$

where $\xi = z/h$; $u_* = (ghS)^{0.5}$

Substituting Eq. (4.69) into Eq. (4.18)

$$\frac{C}{C_a} = A_1 \left[\frac{\xi^{0.5}}{1-(1-\xi)^{0.5}} \right]^{\zeta_\xi} \tag{4.70}$$

$$A_1 = \left[\frac{1-(1-E)^{0.5}}{E^{0.5}} \right]^{\zeta_\xi} \tag{4.71}$$

where $\zeta_\xi = 2w_{ss}/(\beta\kappa u_*)$

The equation of suspended-load becomes

$$g_s = \int_a^h Cudz = C_a h \left(UI_3 - \frac{2u_*}{\kappa} I_4 \right) \tag{4.72}$$

where I_3 and I_4 = integrals are given by

$$I_3 = \left[\frac{1-(1-E)^{0.5}}{E^{0.5}}\right]^{\zeta_\xi} \int_E^1 \left[\frac{\xi^{0.5}}{1-(1-\xi)^{0.5}}\right]^{\zeta_\xi} d\xi \qquad (4.73a)$$

$$I_4 = \left[\frac{1-(1-E)^{0.5}}{E^{0.5}}\right]^{\zeta_\xi} \int_E^1 \left(\frac{\xi}{1-\xi}\right)^{\zeta_\xi} \left\{\ln\left[\frac{\xi^{0.5}}{1-(1-\xi)^{0.5}}\right] - (1-\xi)^{0.5} - \frac{1}{3}\right\}^{\zeta_\xi} d\xi \qquad (4.73b)$$

Similar to Einstein's approach, Eq. (4.72) can be reduced to

$$g_s = \frac{h}{0.8aU}\left(UI_3 - \frac{2u_*}{\kappa} I_4\right) g_b \qquad (4.74)$$

- It was assumed that the velocity of the bed sediment u_b = 0.8U and the thickness of the bed layer is based on **DuBoys**' (1879) assumption

$$a = j \frac{\tau_0 - \tau_c}{(1-\rho_0)(\rho - \rho_s) g \tan\varphi} \qquad (4.75)$$

where j = experimental constant (= 10); ρ_0 = porosity of sediment; τ_c = critical bed shear stress of sediment; and φ = angle of repose

Gravitational Theory of Suspension

Velikanov's Theory

- Principle of energy conservation is applied
- **Velikanov** (1958) expressed the energy balance equations as E_1 = E_3 + E_5 for water phase; and E_2 = E_4 for sediment phase

E_1 and E_2 refer to the amount of energy supplied by the water and sediment phases, E_3 and E_4 denote the energy lost in the water and sediment phases to overcome frictional resistance, and E_5 stands for the amount of energy needed to maintain the suspension

- For two-dimensional uniform flow

$$E_1 = \rho g(1-\overline{C})\overline{u}S \tag{4.76}$$

$$E_2 = \rho g\overline{C}\,\overline{u}S \tag{4.77}$$

$$E_3 = -\overline{u}\frac{d\tau}{dz} = \rho\overline{u}\frac{d}{dz}[(1-\overline{C})\overline{u'w'}] \tag{4.78}$$

$$E_4 = \rho_s \bar{u} \frac{d}{dz}(\bar{C}\,\overline{u'w'}) \quad (4.79)$$

$$E_5 = (\rho_s - \rho) g (1 - \bar{C}) \bar{C} w_{ss} \quad (4.80)$$

where w_{ss} = fall velocity of a sediment particle in still water of infinite extent

- Velikanov assumed the fall velocity of a sediment particle in flowing water is $\bar{w}$ - w_{ss}

- The continuity equation for sediment passing through a unit area located at a distance z from the bed

$$\overline{C(w - w_{ss})} = 0 \quad (4.81)$$

- Continuity equation for water is

$$\overline{w(1 - C)} = 0 \quad (4.82)$$

- If the instantaneous value is expressed as the sum of the time averaged and the fluctuation values

$$\bar{w}\bar{C} - \bar{C} w_{ss} + \overline{w'C'} = 0 \quad (4.83)$$

$$\bar{w} - \bar{C}\bar{w} + \overline{w'C'} = 0 \quad (4.84)$$

Adding Eqs. (4.83) and (4.84) yields

$$\overline{w} = \overline{C} w_{ss} \tag{4.85}$$

- Substituting the related energy terms into the energy balance equations

$$g(1-\overline{C})\overline{u}S = \overline{u}\,\frac{d}{dz}[(1-\overline{C})\overline{u'w'}] + \Delta g(1-\overline{C})\overline{C} w_{ss} \tag{4.86}$$

$$g\overline{C}S = \frac{d}{dz}(\overline{C}\,\overline{u'w'}) \tag{4.87}$$

where $\Delta = s - 1$; and s = relative density of sediment particles, that is ρ_s/ρ

- **Velikanov** suggested the logarithmic law of velocity distribution

$$\overline{u} = \frac{u_*}{\kappa}\ln\left(1+\frac{z}{\Delta_v}\right) = \frac{(ghS)^{0.5}}{\kappa}\ln\left(1+\frac{\xi}{\alpha}\right) \tag{4.88}$$

where Δ_v = parameter depending on the bed roughness; and $\alpha = \Delta_v/h$

Dividing Eq. (4.86) by $\bar{u}$ and adding it to Eq. (4.87)

$$\int_z^h gS dz = \int_z^h \frac{d}{dz}\overline{u'w'}dz + \Delta g w_{ss}\int_z^h \frac{(1-\bar{C})\bar{C}}{\bar{u}}dz \tag{4.89}$$

After integration

$$-gS(h-z) = \overline{u'w'} + \Delta g w_{ss}\int_z^h \frac{(1-\bar{C})\bar{C}}{\bar{u}}dz \tag{4.90}$$

- The second term of the RHS is much smaller than the first term and can be neglected

$$\overline{u'w'} = -gS(h-z) \Rightarrow \frac{d\overline{u'w'}}{dz} = gS \tag{4.91}$$

- For small concentration, 1 – $\bar{C}$ = 1 and the substitution of Eqs. (4.88) and (4.91) into Eq. (4.86) yields the differential equation for concentration distribution

$$\frac{dC}{C} = \beta_v \frac{d\xi}{(1-\xi)\ln[1+(\xi/\alpha)]} \tag{4.92}$$

where $\beta_v = \Delta\kappa w_{ss}/[S(ghS)^{0.5}]$

- Vertical distribution of sediment concentration is obtained from Eq. (4.92)

$$\frac{C}{C_\alpha} = \exp(-\beta_v \zeta_v) \qquad (4.93)$$

$$\zeta_v = \int_\alpha^\xi \frac{d\xi}{(1-\xi)\ln[1+(\xi/\alpha)]} \qquad (4.94)$$

- Shortcoming of the gravitational theory is that the energy balance equation is not scientifically sound
- Energy for suspension E_5 comes from the energy of turbulence that functions as the energy loss of the flow in order to overcome resistance
- In energy balance equation, that part of the dissipated energy should not be taken into account two times

Sediment Suspended-load by Gravitational Theory

Velikanov's Approach

- **Velikanov** (1958) assumed the sediment concentration is small, that is 1 – $\overline{C}$ = 1, and integrated Eq. (8.86) over the flow depth

$$\int_0^h g\bar{u}S\,dz = \int_0^h \bar{u}\frac{d}{dz}\overline{u'w'}\,dz + \int_0^h \Delta g\overline{C}w_{ss}\,dz \qquad (4.95)$$

- In the above, $-\overline{u'w'} = \tau/\rho$. Since $\tau \sim U^2$

$$\int_0^h \bar{u}\frac{d}{dz}\overline{u'w'}\,dz = bU^3 \qquad (4.96)$$

Eq. (4.95) is integrated, simplified using Eq. (4.96)

$$\frac{b}{\lambda} + \Delta\frac{\overline{C}_{av}w_{ss}}{US} = 1 \qquad (4.97)$$

where $\lambda = ghS/U^2$; and $\overline{C}_{av}$ = depth-averaged concentration

- For clear water flow, $\overline{C}_{av}$ = 0, and from Eq. (4.97)

$$b = \lambda_0 \qquad (4.98)$$

- For maximum sediment carrying capacity of the flow

$$\lambda = \lambda_k \qquad (4.99)$$

- The value of the ratio λ_0/λ_k is approximately taken as constant. Substituting this ratio into Eq. (4.97) yields

$$\Delta \frac{\overline{C}_{av} w_{ss}}{US} = 1 - \frac{\lambda_0}{\lambda_k} \qquad (4.100)$$

$\overline{C}_{av}$ is considered to be saturated depth-averaged concentration

- The depth-averaged velocity can be given by

$$U = \frac{1}{h}\int_0^h \overline{u}\,dz = \int_0^h \frac{(ghS)^{0.5}}{\kappa} \ln\left(1 + \frac{z}{\Delta_v}\right) dz = c_1 \frac{(ghS)^{0.5}}{\kappa} \qquad (4.101)$$

where c_1 = (1+α)[ln(1+α) – 1] = $f(\alpha)$

Substituting Eq. (4.101) into Eq. (4.100) yields

$$\Delta \frac{\kappa \overline{C}_{av} w_{ss}}{c_1 S (ghS)^{0.5}} = 1 - \frac{\lambda_0}{\lambda_k} \tag{4.102}$$

The above equation is therefore given by $\beta_v \overline{C}_{av} / c_1 = 1 - \lambda_0 / \lambda_k =$ constant

$$\overline{C}_{av} \sim \frac{c_1}{\beta_v} = \frac{\kappa^2 U^3}{\Delta f^2(\alpha) g h w_{ss}} \tag{4.103}$$

The general form of the above equation

$$\overline{C}_{av} = K \frac{U^3}{g h w_{ss}} \tag{4.104}$$

where K = constant to be determined experimentally

- Researchers of Wuhan Institute of Hydraulic and Electric Engineering (**WIHEE** 1961) made an extensive analysis of field data and concluded that Eq. (4.104) should be modified as

$$\overline{C}_{av} = K \left(\frac{U^3}{g h w_{ss}} \right)^{m_1} \tag{4.105}$$

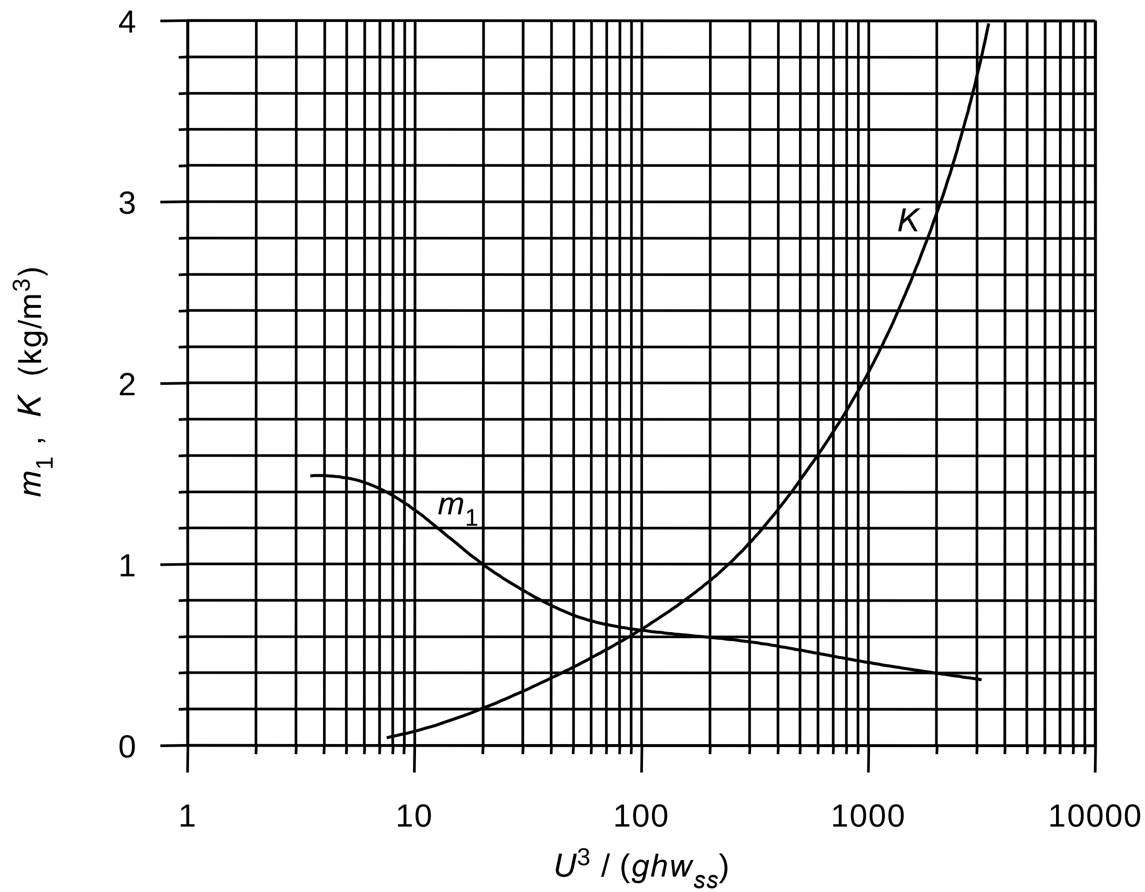

Fig. 4.5 Variations of K (in kg/m^3) and m_1 with $U^3/(ghw_{ss})$

Bagnold's Model for Suspended-load Transport

- **Bagnold** (1966) investigated the suspended-load transport using the same method that he used for the bed-load transport

- The rate of suspended sediment load g_{ss} (in submerged weight) can be expressed as

$$g_{ss} = W_S \bar{u}_s \qquad (4.106)$$

where $\bar{u}_s$ = depth-averaged velocity of the suspended-load, and W_S = total submerged weight of suspended sediment in the column

- The amount of flow potential energy used to sustain the bed-load motion equals the work done for sediment suspension can be expressed as

$$W_S w_{ss} = \tau_0 U (1 - e_b) e_s \qquad (4.107)$$

where e_b and e_s = efficiencies for bed-load and suspended-load transport

- Combining Eqs. (4.106) and (4.107)

$$g_{ss} = \tau_0 U \frac{\bar{u}_s}{w_{ss}}(1 - e_b)e_s \tag{4.108}$$

- Since suspended sediments move with the same velocity as the flow

$$\bar{u}_s = \frac{1}{h - a}\int_a^h Cudz \tag{4.109}$$

Here *a* refers to the lower boundary of the suspension zone to the bed

- Since the velocity increases and the sediment concentration decreases with *z*, $\bar{u}_s$ is generally smaller than depth-averaged velocity *U*

- If $r = \bar{u}_s/U < 1$, then Eq. (4.108) can be written

$$g_{ss} = \tau_0 U \frac{U}{w_{ss}} r(1 - e_b)e_s \tag{4.110}$$

- The suspended load g_s (in weight) transport rate is

$$g_s = \tau_0 U \frac{sU}{\Delta w_{ss}} r(1 - e_b)e_s \tag{4.111}$$

- **Bangnold** reviewed the laboratory data and obtained $r(1 - e_b)e_s$ = 0.01
- The suspended-load rate is

$$g_s = 0.01\tau_0 U \frac{sU}{\Delta w_{ss}} \tag{4.112}$$

Mixing-Length Model for Suspended-Load Transport

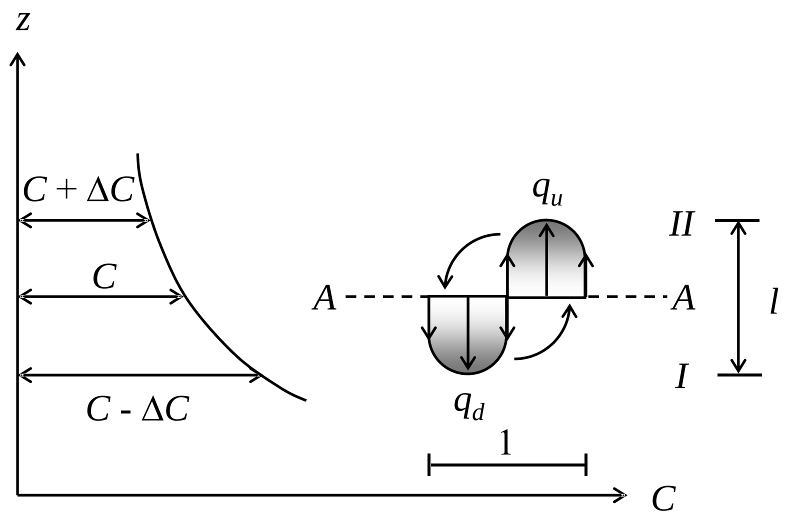

Fig. 4.6 Sediment suspension in turbulent flow

- Following the concept Prandtl's mixing length theory, fluid and sediment are transported from lower level *I* where the (volumetric) concentration of suspended sediment is C - ΔC up to a height level *II* where the concentration is $C + \Delta C$

- The fluid (volume per unit time and area) transfers up through the section AA with the amount of sediment q_u

$$q_u = (w' - w_{ss})\left(C - \frac{l}{2} \cdot \frac{dC}{dz} \right) \qquad (4.113)$$

- Analogous to Eq. (4.113), the downward sediment transport q_d is

$$q_d = (w' + w_{ss})\left(C + \frac{l}{2}\cdot\frac{dC}{dz}\right) \tag{4.114}$$

- In case of a steady flow situation, q_u and q_d are equal

$$Cw_{ss} + \frac{w'l}{2}\cdot\frac{dC}{dz} = 0 \tag{4.115}$$

- By assuming $w'l/2 \approx \beta\varepsilon_s l$ and using Eq. (4.23)

$$Cw_{ss} + \kappa\beta u_* z\left(1 - \frac{z}{h}\right)\frac{dC}{dz} = 0 \tag{4.116}$$

- Integrating, the vertical distribution of the concentration is obtained

$$\frac{C}{C_a} = \left(\frac{h-z}{z}\cdot\frac{a}{h-a}\right)^{\zeta_\beta} \tag{4.117}$$

Total-Load Transport

- The amount of sediment that passes through a given river reach for given conditions of the flow and boundary is termed *total-load*
- Total-load is the sum of the bed-load and suspended-load
- Two general approaches to determine the total-load
- Separate estimation of bed-load and suspended-load
- Determination of the total load function directly without dividing it into bed-load and suspended-load

Indirect Estimation of Total-Load Transport

Einstein's Approach

Einstein (1950) advanced the bed-load and the suspended-load concept for the estimation of total-load

- The total-load transport g_t of a given size fraction i_t is

$$i_t g_t = i_b g_b + i_s g_s \qquad (4.118)$$

where g_b and g_s = bed-load and suspended-load transport rates, respectively; and i_b and i_s = particle size fractions of bed-load and suspended-load transport rates

- Using Eq. (4.64) into Eq. (4.118), the total-load transport g_t of a given size fraction i_t is

$$i_t g_t = i_b g_b (1 + P_E I_1 + I_2) \qquad (4.119)$$

Bagnold's Modified Approach

- **Bagnold** (1966) considered the relationship between the rate of energy available to a fluvial system and the rate of work done by the system in transporting sediment

$$g_b = \frac{\tau_0 s}{\Delta \tan \varphi} U e_b \qquad (4.120)$$

where τ_0 = bed shear stress; Δ = s – 1; s = relative density of sediment particles, that is ρ_s/ρ; ρ_s = mass density of sediment; ρ = mass density of fluid; φ = angle of repose; and U = depth-averaged flow velocity; and e_b = efficiency for bed-load transport

- Using the expression of suspended-load transport rate g_s, the total-load transport of g_t (= g_b + g_s) is

$$g_t = \frac{\tau_0 s U}{\Delta}\left(\frac{e_b}{\tan \varphi} + 0.01\frac{U}{w_{ss}}\right) \qquad (4.121)$$

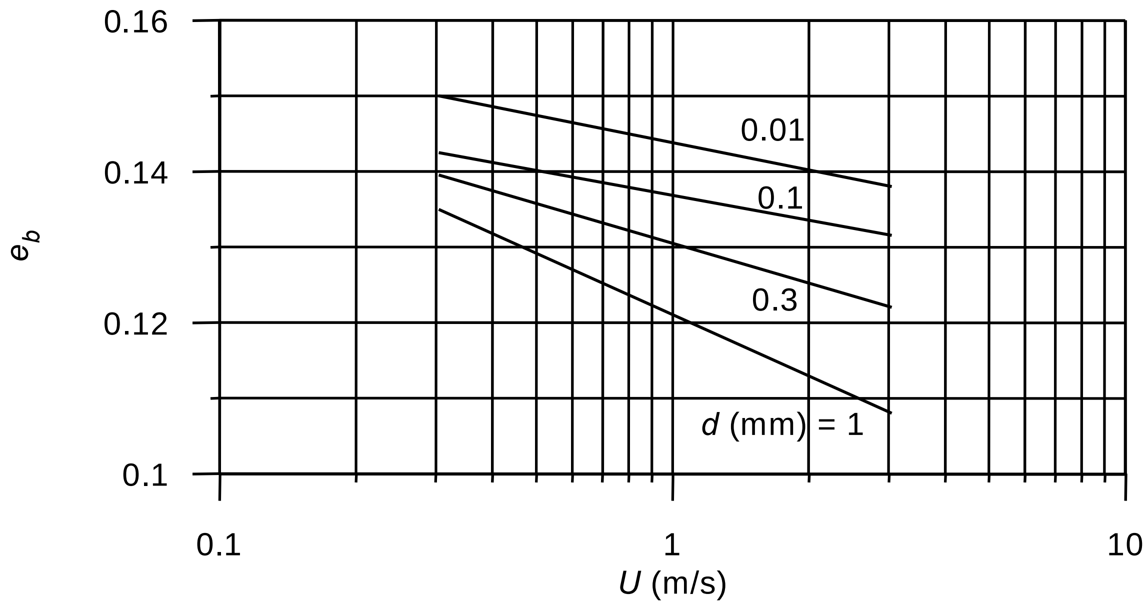

Fig. 4.7 Variation of bed-load transport efficiency e_b with U for different particle size d

Direct Estimation of Total-Load Transport

Graf and Acaroglu Approach

- **Graf and Acaroglu** (1968) used hydraulic radius R_b to develop a *shear intensity parameter* ψ_A as transport criterion

$$\Psi_A = \frac{\Delta d}{S R_b} \qquad (4.122)$$

- Based on a work rate concept, a *transport parameter* was established

$$\Theta_A = \frac{\overline{C} U R_b}{(\Delta g d^3)^{0.5}} \qquad (4.123)$$

where $\overline{C}$ = volumetric concentration of the transported particles

- Using experimental data of different investigators, **Graf and Acaroglu** (1968) obtained the following empirical relationship between Θ_A and ψ_A

$$\Theta_A = \frac{10.39}{\Psi_A^{2.52}} \qquad (4.124)$$

Calculations

Calculation of Hydraulic Parameters:

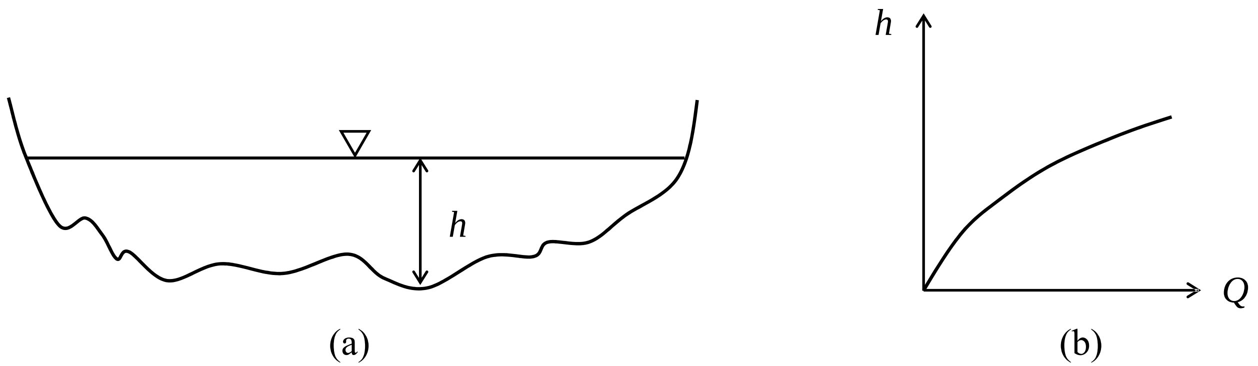

Fig. 4.8 (a) Schematic of a channel section and (b) stage discharge curve

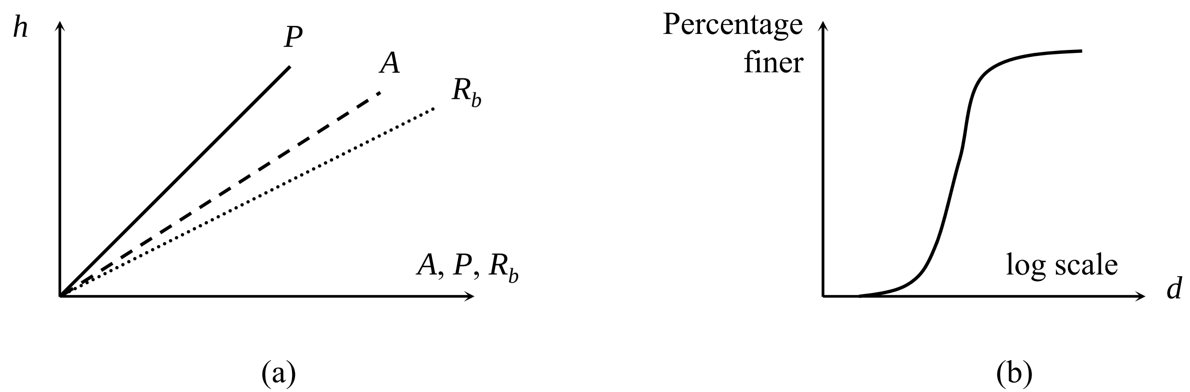

Fig. 4.9 (a) Channel characteristics curves and (b) particle size distribution curve

1. For a given channel section, the stage discharge curve (h versus Q), the channel characteristics curves (h versus area A, wetted perimeter P and hydraulic radius R_b) and the particle size distribution curve are given. The streamwise bed slope of the channel S is also known

2. Assume different values of R_b' to cover the entire discharge Q_{max}

3. Calculate $u_*' = (g R_b' S)^{0.5}$

4. Calculate $\delta = 11.6\nu / u_*'$

5. Find $k_s = d_{65}$ from particle size distribution curve

6. Find x from Fig. 3.3 (curve x versus k_s/δ)

7. Calculate $\Delta_k = k_s/x$

8. Calculate $U = u_*' \, 5.75 \log (12.27 R_b' / \Delta_k)$

9. Calculate $\Psi = \Delta d_{65}/(R_b' S)$

10. Find U/u_*'' from Fig. 4.10 (curve U/u_*'' versus Ψ)

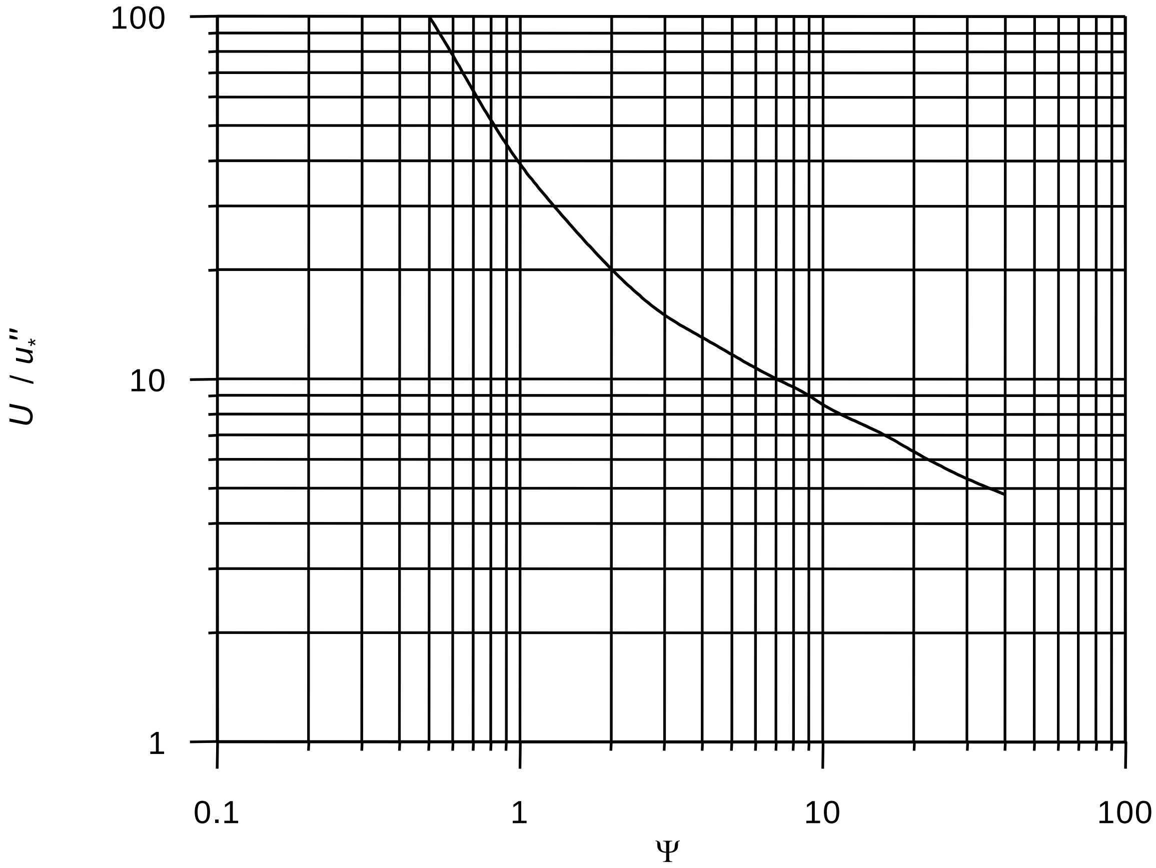

Fig. 4.10 Dependency of U/u''_* on Ψ

11. Determine u''_* (shear velocity due to channel irregularities such as bed forms)

12. Calculate R''_b from u''_* = $(g R''_b S)^{0.5}$

13. Calculate $R_b = R'_b + R''_b$

14. Calculate $u_* = (gR_bS)^{0.5}$

15. Find flow depth *h* form channel characteristics curves

16. Find flow area *A* form channel characteristics curves

17. Find wetted perimeter *P* form channel characteristics curves

18. Estimate flow discharge $Q = UA$

19. Determine the characteristic distance X: $X(\Delta_k/\delta > 1.8) = 0.77\Delta_k$ and $X(\Delta_k/\delta < 1.8) = 0.77\delta$

20. Determine the lift correction factor Y from Fig. 3.5

21. Calculate $\beta_x = \log(10.6X/\Delta_k)$

22. Evaluate $(\beta/\beta_x)^2$, with $\beta = \log(10.6)$

23. Calculate Einstein's transport parameter P_E: $P_E = 2.303\log(30.2h/\Delta_k)$

Calculation of Total Load:

24. The representative particle size *d* is know from the particle size distribution curve given in Fig. 4.9(b)

25. The corresponding fraction i_b is obtained from the ordinate scale of Fig. 4.9(b)

26. For d/X, find the hiding factor ξ from Fig. 3.4

27. Calculate $\Psi_* = \xi Y \Psi(\beta/\beta_x)^2$

28. Find Φ from Fig. 3.6 (curve Ψ_* versus Φ)

29. Calculate $i_b g_b = i_b \Phi \Delta^{0.5} \rho_s g^{1.5} d^{1.5}$

30. Calculate $i_b G_b = (i_b g_b)P$, bed-load rate in weight per unit time for a size fraction for entire cross section

31. Calculate $\sum i_b G_b$, bed-load rate in weight per unit time for all size fractions for entire cross section

32. Calculate $E = a/h$ with $a = d_{65}$

33. Calculate $\zeta = w_{ss}/(\kappa\, u'_*)$

34. Find I_1 and I_2 from Eqs. (4.60a) and (4.60b) by numerical integration

35. Calculate $i_t g_t = (1 + P_E I_1 + I_2)$

36. Calculate $i_t G_t = (i_t g_t)P$, total-load rate in weight per unit time for a size fraction for entire cross section

37. Calculate $\sum i_t G_t$, total-load rate in weight per unit time for all size fractions for entire cross section

Thank You