SEDIMENT THRESHOLD

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- When a stream flows over a loose sedimentary bed, hydrodynamic forces are exerted upon the sediment particles at the bed surface
- Increase in flow velocity causes an increase in the magnitude of hydrodynamic forces
- Sediment particles start to move if a situation is eventually reached when the hydrodynamic forces induced by the flow exceed a certain limiting value
- Initial movement of sediment particles is frequently called *incipient motion*
- The condition being just sufficient to initiate sediment motion is termed *threshold or critical condition*
- Threshold of sediment motion in open channels having erodible bed is an important component of management of river systems

Sediment Properties

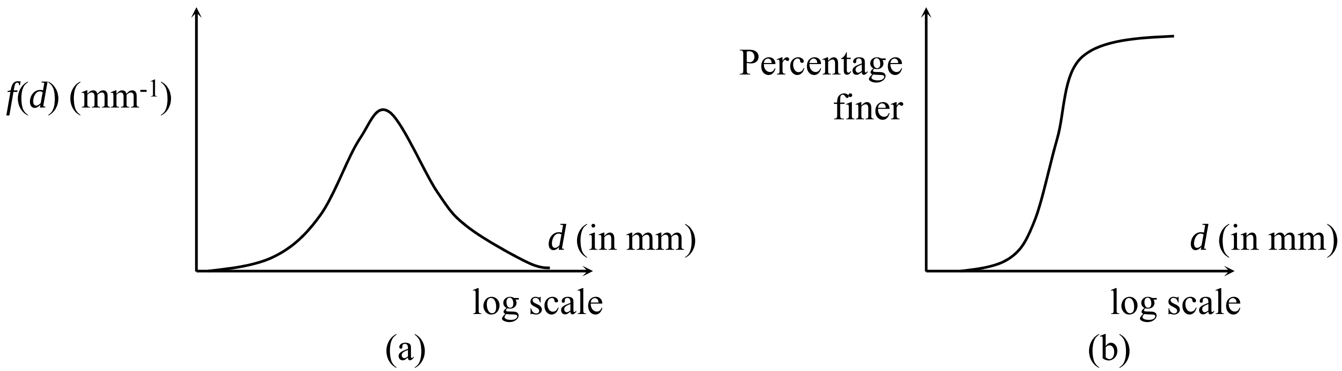


Fig. 2.1 (a) Frequency curve and (b) cumulative frequency curve

- Often the distribution curve of sediments approaches the log-normal probability curve when plotted as in Fig. 2.1(a)
- Distribution function is log-normal and is given by

$$f(d) = \frac{1}{\sqrt{2\pi} d \ln(\sigma_g)} \exp\left\{ -0.5 \left[\frac{\ln(d/d_{50})}{\ln(\sigma_g)} \right]^2 \right\} \tag{2.1}$$

where σ_g = geometric standard deviation, given by $(d_{84}/d_{16})^{0.5}$; d_{50} = median particle diameter or 50 percent finer (by weight) particle diameter. For uniform sediments, $\sigma_g < 1.4$

Definitions of Sediment Threshold

First type of definition is based on sediment flux

- **Shields** (1936) put forward a concept of sediment threshold that the bed shear stress has a value for which the extrapolated sediment flux becomes zero
- **USWES** (1936) set a concept of sediment threshold that the tractive force brings about *general motion* of bed particles. For sediment particles less than 0.6 mm, this concept was found to be inadequate and general motion was redefined that sediment in motion should reasonably be represented by all sizes of bed particles and that sediment flux should exceed 4.1×10^{-4} kg /sm

Second type of definition is based on bed particle motion

- **Kramer** (1935) indicated four different bed shear conditions for sedimentary bed
  - No particles are in motion, termed *no transport*
  - A few of the smallest particles are in motion at isolated zones, termed *weak transport*
  - Many particles of mean size are in motion, termed *medium transport*
  - Particles of all sizes are in motion at all points and at all times, termed *general transport*
- **Kramer** (1935) pointed out the difficulty of setting up clear limits between these regimes but defined threshold bed shear stress to be that stress initiating general transport
- **Vanoni** (1964) proposed that the sediment threshold is the condition of particle motion in every two seconds at any bed position

Competent Velocity Concept

- A competent bed velocity or competent mean velocity is a velocity at particle level or mean velocity, which is just enough to move the particles of a given size

- **Goncharov** (1964) defined threshold velocity as detachment velocity U_n, which was defined as the lowest average velocity at which individual particles continually detaches from the bed for which the mean value of the fluctuating lift force nearly equals the submerged weight of particle in water

$$U_n = \log(8.8h/d)\sqrt{0.57\Delta gd} \qquad (2.2)$$

where h = flow depth; d = representative particle diameter, that is median particle diameter; g = acceleration due to gravity; $\Delta = s - 1$; s = relative density of sediment particles, that is ρ_s/ρ; ρ_s = mass density of sediment; and ρ = mass density of fluid

- **Carstens** (1966) reported an equation of critical or threshold velocity u_{cr} at the particle level having analyzed a large number of published data on threshold of sediment motion as

$$u_{cr}^2 / \Delta g d \approx 3.61(\tan\varphi \cos\theta - \sin\theta) \qquad (2.3)$$

where φ = angle of repose of sediment; and θ = angle made by the streamwise sloping bed with the horizontal

- **Neill** (1968) presented a conservative design curve for the movement of coarse uniform gravel in terms of average threshold velocity U_{cr} and represented it in an equation

$$U_{cr}^2 / \Delta g d = 2(h / d)^{1/3} \qquad (2.4)$$

- **Zanke** (1977) proposed the following equation

$$U_{cr} = 2.8\sqrt{\Delta g d} + 14.7 c_1 \nu / d \qquad (2.5)$$

where c_1 = a coefficient for cohesiveness varying from 1 for non-cohesive to 0.1 for cohesive sediments

- Many researchers have validly criticized the use of critical velocity equation as a criterion for threshold of sediment motion
- Confusion regarding the competent velocity at particle level u_{cr} and average velocity for threshold condition U_{cr}
- Hydraulicians accept a more satisfactory quantity, the bed shear stress as a sediment threshold
- **Yang** (1973) developed a promising model for the estimation of average velocity for sediment threshold

Yang's Competent Velocity Model

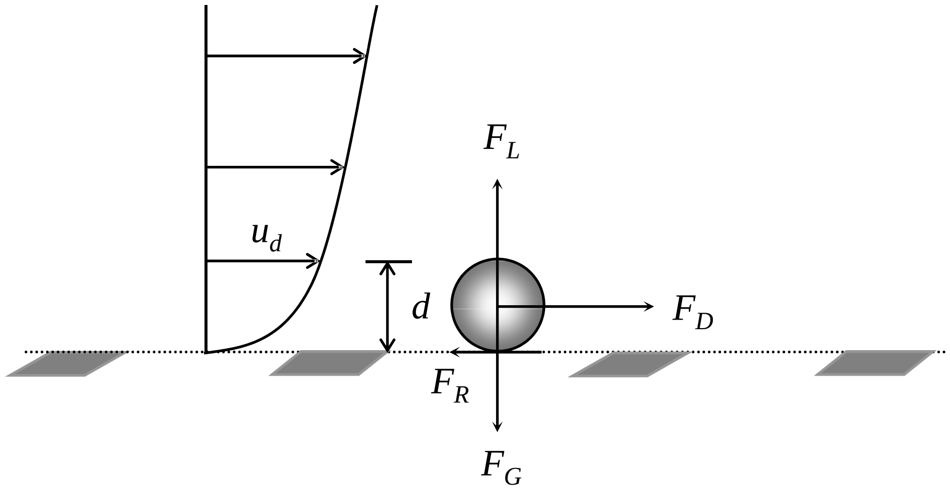


Fig. 2.2 Forces acting on a spherical sediment particle at the bottom of an open channel

Yang (1973) proposed a model on competent velocity

- The drag force F_D is expressed as

$$F_D = C_D \frac{\pi}{8} d^2 \rho u_d^2 \qquad (2.6)$$

where C_D = drag coefficient; and u_d = velocity at a distance d above the bed

- The terminal fall velocity w_{ss} of a spherical particle is reached when there is a balance between the drag force F_D and submerged weight F_G of the particle

$$C_{D1}\frac{\pi}{8}d^2\rho w_{ss}^2=\frac{\pi}{6}d^3(\rho_s-\rho)g\ (=F_G) \qquad (2.7)$$

where C_{D1} = drag coefficient at w_{ss}, assumed as $\psi_1 C_D$

$$F_D=\frac{\pi}{6\psi_1 w_{ss}^2}d^3(\rho_s-\rho)gu_d^2 \qquad (2.8)$$

- Considering the logarithmic law for velocity distribution, velocity at particle level u_d and depth-averaged velocity U are as follows

$$u_d=B_r u_* \qquad (2.9a)$$

$$U=u_*\left[5.75\left(\log\frac{h}{d}-1\right)+B_r\right] \qquad (2.9b)$$

where B_r = roughness function; and u_* = shear velocity

Using Eqs. (2.9a) and (2.9b) into Eq. (2.8), yields

$$F_D = \frac{\pi}{6\psi_1} d^3 (\rho_s - \rho) g \left(\frac{U}{w_{ss}} \right)^2 \frac{B_r^2}{\left[5.75 \left(\log \frac{h}{d} - 1 \right) + B_r \right]^2} \tag{2.10}$$

- The lift force F_L acting on the particle is given by

$$F_L = C_L \frac{\pi}{8} d^2 \rho u_d^2 \tag{2.11}$$

where C_L = lift coefficient, assumed as C_D/ψ_2

Using Eqs. (2.9a) and (2.9b) into Eq. (2.11), yields

$$F_L = \frac{\pi}{6\psi_1\psi_2} d^3 (\rho_s - \rho) g \left(\frac{U}{w_{ss}} \right)^2 \frac{B_r^2}{\left[5.75 \left(\log \frac{h}{d} - 1 \right) + B_r \right]^2} \tag{2.12}$$

- The drag force F_D is balanced by the resistance force F_R

$$F_D = F_R = \psi_3 (F_G - F_L) \qquad (2.13)$$

where ψ_3 = friction coefficient

Inserting Eqs. (2.7), (2.10) and (2.12) in Eq. (2.13), one gets the equation of average critical or threshold velocity U_c

$$\frac{U_c}{w_{ss}} = \sqrt{\frac{\psi_1 \psi_2 \psi_3}{\psi_2 + \psi_3} \left[\frac{5.75}{B_r} \left(\log \frac{h}{d} - 1 \right) + 1 \right]} \qquad (2.14)$$

- **Yang** (1973) gave equations for both smooth and rough boundaries

$$\frac{U_c}{w_{ss}} = \frac{2.5}{\log R_* - 0.06} + 0.66 \qquad \text{for } 0 < R_* < 70 \qquad (2.15)$$

$$\frac{U_c}{w_{ss}} = 2.05 \qquad \text{for } R_* \geq 70 \qquad (2.16)$$

Lift Force Concept

- Lift force may arise for two reasons
  - Pressure difference due to a steep velocity gradient at the bottom of channel
  - Upward velocity component adjacent to the bed as a result of turbulence
- **Jeffreys** (1929) showed that the classical-hydrodynamics provides a simple explanation of lifting and carrying solid particles in fluid
- Assuming a potential flow over a circular cylinder with its major axis perpendicular to the flow, lift takes place if

$$(3+\pi^2)U^2 > 9\Delta g r_1 \qquad (2.17)$$

where r_1 = radius of the cylinder

- The shortcoming of Jeffreys model is that the drag forces are totally discarded

- **Reitz** (1936) discussed a similar idea and suggested to express the beginning of sediment motion with a lift model

- **Lane and Kalinske** (1939) stressed on turbulence for determination of lift and assumed

  - Particles having a settling velocity smaller than the instantaneous turbulent fluctuations at bed experience lift

  - Velocity fluctuations vary according to the normal error law

  - Turbulent fluctuations and shear velocities are related

- **White** (1940) carried out a single experiment and found that the lift on an individual particle is very small compared to its weight

- **Einstein and El-Samni** (1949) measured the lift force directly as a pressure difference

$$f_L = 0.5 C_L \rho u_{0.35d}^2 \qquad (2.18)$$

where f_L = lift force per unit area of the particle; C_L = lift coefficient assumed as 0.178; and $u_{0.35d}$ = measured velocity of flow at a distance of 0.35 diameter (equivalent) from the theoretical wall

- **Iwagaki** (1956) worked on the problem of sediment threshold using shear stress concept
- The results of the study of **Einstein and El-Samni** (1949) were used by **Task Committee** (1966), who calculated f_L/τ_c; where τ_c = threshold bed shear stress
- **Chepil** (1961) pointed out that, once the particle is displaced, lift force tend to diminish and drag force to increase
- **Coleman** (1967) studied the lift forces acting on a sphere placed on a hypothetical streambed and observed negative lift force for Reynolds number less than 100

Threshold Shear Stress Concept

Empirical Equations of Threshold Shear Stress

- **Kramer** (1935) carried out experiments in a flume using quartz particles of relative density 2.7

$$\tau_c = 29\sqrt{(\rho_s - \rho) g\, d / M} \qquad (2.19)$$

where where τ_c = threshold or critical bed shear stress; and *M* = uniformity coefficient of Kramer

- **USWES** (1936) recommended the formula

$$\tau_c = 0.285\sqrt{\Delta d / M} \qquad (2.20)$$

- **Leliavsky** (1955) represented the threshold bed shear stress with a simple relationship as

$$\tau_c = 166\, d \qquad (2.21)$$

Theoretical and Semi-Theoretical Analyses

Shields Diagram

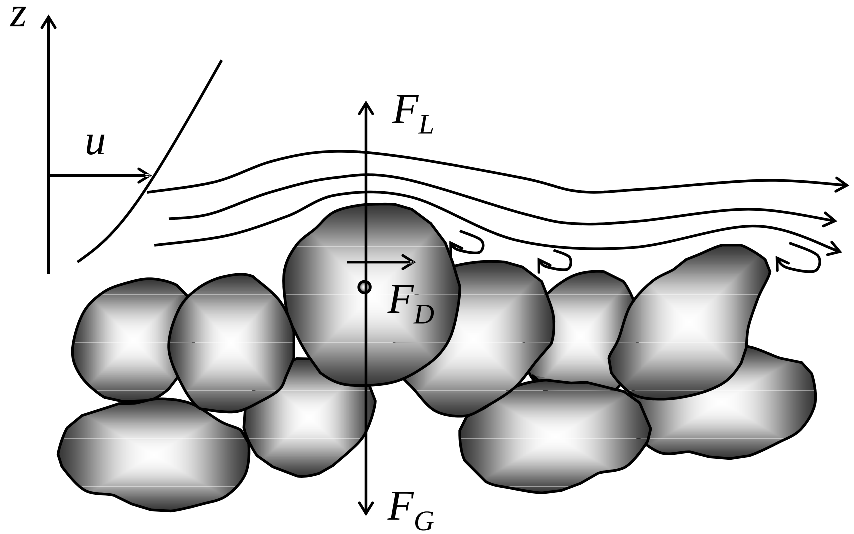


Fig. 2.3 Forces acting on a sediment particle resting on bed

Shields (1936) was pioneer to present a semi-theoretical theory

- The driving force is the flow drag force F_D exerted on the sediment particle

$$F_D = C_D \frac{1}{2}\rho u^2 A = f_1\left(a_1, \frac{ud}{\nu}\right)\rho d^2 u^2 \qquad (2.22)$$

where u = velocity at elevation $z = a_2 d$; A = frontal area of the particle; and a_1 = particle shape factor

- Velocity distributions for flow over rough and smooth boundaries have the form

$$\frac{u}{u_*} = 5.75 \log \frac{z}{k_s} + \frac{z u_*}{\nu} = 5.75 \log a_2 + f_2\left(\frac{u_* d}{\nu}\right) \quad (2.23)$$

where k_s = roughness height being proportional to d

- Drag force is

$$F_D = \tau_0 d^2 f_3(a_1, a_2, R_*) \quad (2.24)$$

- The resistance to motion F_R was assumed to be dependent only upon the roughness of the bed and the submerged weight F_G of the particle

$$F_R = a_3 \Delta\rho g d^3 \quad (2.25)$$

where a_3 = roughness factor

- At the incipient condition, when the sediment particle is about to move, $u_* \rightarrow u_{*c}$ (that is critical shear velocity), then drag force is balanced by the resistance

$$F_D = F_R \quad (2.26)$$

Rearranging the terms

$$\frac{u_{*c}^2}{\Delta g d} = \frac{\tau_c}{\Delta \rho g d} = f(R_*) \qquad (2.27)$$

- The Shields parameter Θ is defined as

$$\Theta = \frac{u_*^2}{\Delta g d} \qquad (2.28)$$

- The Shields parameter Θ is defined as

$$\Theta_c = f(R_*) \qquad (2.29)$$

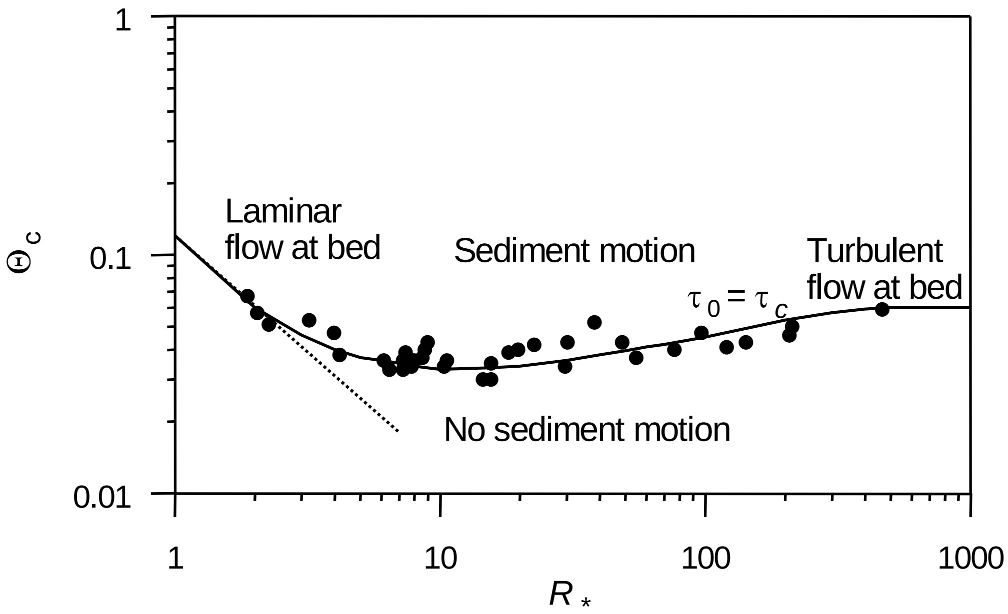


Fig. 2.4 Shields parameter Θ as a function of particle Reynolds number R_*

- The Shields diagram has three distinct zones:
  - Hydraulically smooth flow for $R_* \leq 2$: d is much smaller than the thickness of viscous sub-layer; and it was found that $\Theta_c = 0.1/R_*$
  - Hydraulically rough flow for $R_* \geq 500$: The viscous sub-layer does not exist. The critical Shields parameter Θ_c is independent of the fluid viscosity and has a constant value of 0.06
  - Hydraulically transitional flow for $2 \leq R_* \leq 500$: Sediment particles are of the order of the thickness of viscous sub-layer. There is a minimum value of Θ_c of 0.032 corresponding to $R_* = 10$

Drawback of the Shields theory are as follows:

- The viscous sub-layer does not have any effect on the velocity distribution when $R_* \geq 70$, but his diagram shows that Θ_c still varies with R_* when the latter is greater than seventy
- Shields used bed shear stress and shear velocity in his diagram as dependent and independent variables, which is not appropriate as they are interchangeable
- Threshold bed shear stress to be determined through trial and error method

- **van Rijn** (1984) gave the empirical equations of the Shields curve

$$\Theta_c(D_* \le 4) = 0.24 / D_* \qquad (2.30a)$$

$$\Theta_c(4 < D_* \le 10) = 0.14 / D_*^{0.64} \qquad (2.30b)$$

$$\Theta_c(10 < D_* \le 20) = 0.04 / D_*^{0.1} \qquad (2.30c)$$

$$\Theta_c(20 < D_* \le 150) = 0.013 D_*^{0.29} \qquad (2.30d)$$

$$\Theta_c(D_* > 150) = 0.055 \qquad (2.30e)$$

where D_* = particle parameter, that is $d(\Delta g/\nu^2)^{1/3}$

- **Julien** (1998) proposed the empirical equations of the Shields curve

$$\Theta_c(D_* \le 0.3) = 0.5 \tan\varphi \qquad (2.31a)$$

$$\Theta_c(0.3 < D_* \le 19) = 0.25 \tan\varphi / D_*^{0.6} \qquad (2.31b)$$

$$\Theta_c(19 < D_* \le 50) = 0.013 \tan\varphi D_*^{0.4} \qquad (2.31c)$$

$$\Theta_c(D_* > 50) = 0.06 \tan\varphi \qquad (2.31d)$$

White's Analysis

- If one neglects the lift force, at limiting equilibrium, the drag force (shear drag) is balanced by the frictional resistance

High-Speed Case ($R_* \geq 3.5$):

- High flow velocity is required to move larger sediment particles, where the drag due to skin friction is negligible as compared to the drag due to pressure difference. If p_f is the packing coefficient defined by Nd^2, where N is the number of particles per unit area, the shear drag per particle (that is τ_0/N) is given by $\tau_0 d^2/ p_f$
- At limiting equilibrium of a particle resting on a horizontal bed, the shear drag is balanced by the product of submerged weight of the particle and the frictional coefficient $\tan\varphi$

$$\Theta_c = \frac{\pi}{6} p_f \tan\varphi \qquad (2.32)$$

- **White** (1940) introduced *turbulence factor* T_f, which is the ratio of the instantaneous bed shear stress to the mean bed shear stress

$$\Theta_c = \frac{\pi}{6} p_f T_f \tan\varphi \qquad (2.33)$$

Low-Speed Case ($R_* < 3.5$):

- Upper portion of the particle is exposed to the shear drag that acts above the center of gravity of the particle
- Effect is taken into account introducing a coefficient α_f

$$\Theta_c = \frac{\pi}{6} p_f \alpha_f \tan\varphi \qquad (2.34)$$

- He experimentally obtained $p_f \alpha_f = 0.34$ as an average value

Wilberg and Smith Approach

- On a horizontal bed, the expression for the force balance equation given by **Wiberg and Smith** (1987)

$$(F_G - F_L)\tan\varphi = F_D \tag{2.35}$$

- The submerged weight of the particle F_G, drag force F_D and lift force F_L are as follows

$$F_G = \Delta\rho g V \tag{2.36}$$

$$F_D = C_D \frac{1}{2}\rho u^2 A_x = C_D \frac{1}{2}\tau_0[f^2(z/z_0)]A_x \tag{2.37}$$

$$F_L = C_L \frac{1}{2}\rho(u_T^2 - u_B^2)A_x = C_L \frac{1}{2}\tau_0[f^2(z_T/z_0) - f^2(z_B/z_0)]A_x \tag{2.38}$$

where V = volume of particle; A_x = frontal area of particle; u = velocity at z above bed; z_0 = zero-velocity level; u_T = velocity at top of particle; u_B = velocity at bottom of particle; z_T = height of top point of particle from bed; and z_B = height of bottom point of particle from bed

- Assuming the bed level is passing through the mid points (those are the contact points) of the bed particles

Using Eqs. (2.36) – (2.38), the following expression for Θ_c is obtained

$$\Theta_c = \frac{2}{C_D \alpha_0} \cdot \frac{1}{f^2(z/z_0)} \cdot \frac{\tan\varphi}{1 + (F_L/F_D)_c \tan\varphi} \qquad (2.39)$$

where $\alpha_0 = A_x d/V$

- C_D is a function of particle Reynolds number (Schlichting 1960), $C_L = 0.2$ and $\cos\varphi = [(d/k_s) + z_*]/[(d/k_s) + 1]$
- For natural sands, $z_* = -0.02$
- For smooth regime ($R_* < 3$) and transitional regime ($3 \leq R_* < 100$), **Reichardt**'s (1951) equation of velocity distribution was used
- For rough regime ($R_* \geq 100$), universal logarithmic velocity distribution was used

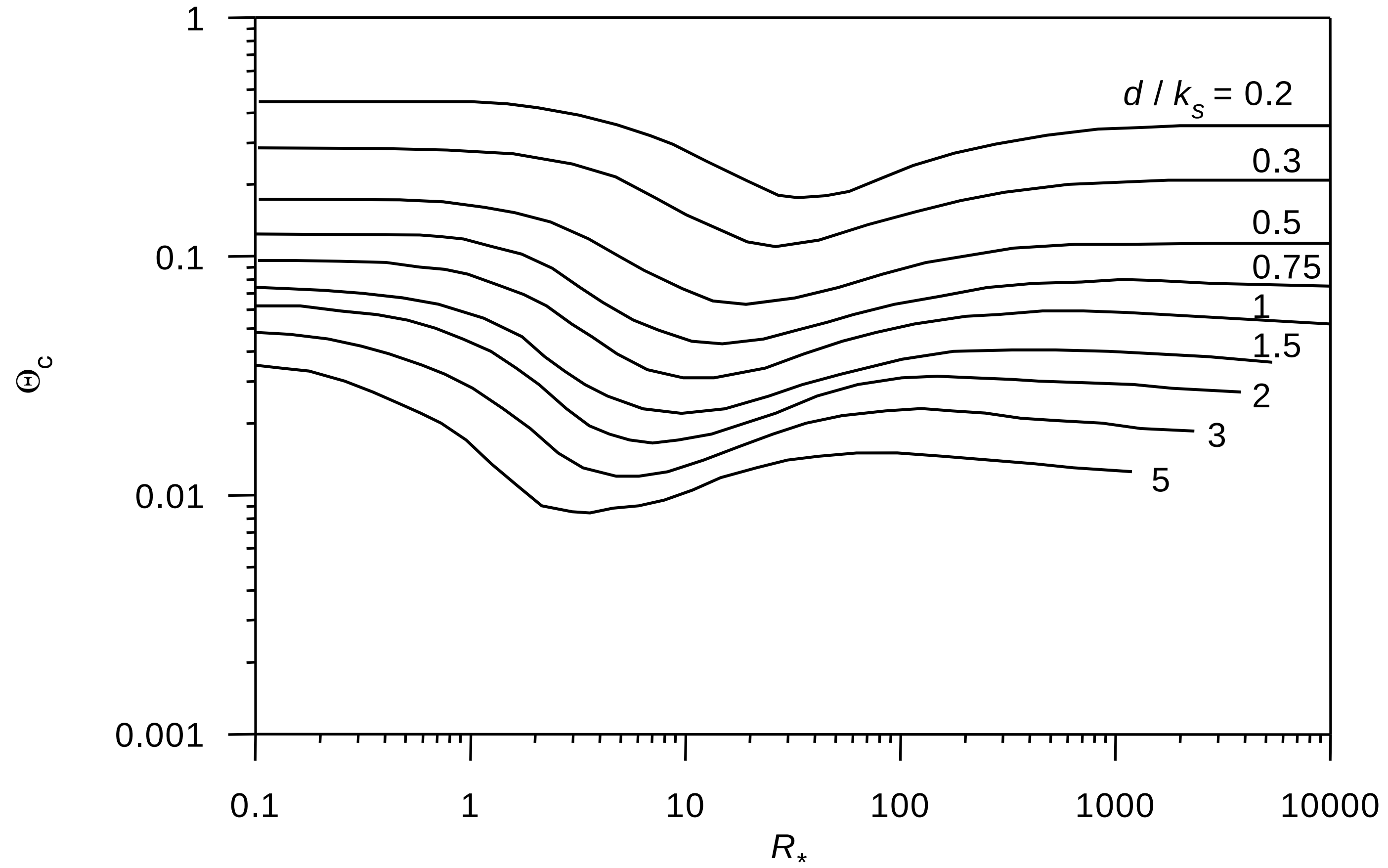


Fig. 2.5 Θ_c as a function of R_* for different d/k_s

Equations of Other Investigators

- **Kurihara** (1948) considered the bed shear stress to be a sum of time-averaged bed shear stress due to main flow and bed shear stress resulting from turbulent velocity fluctuations

- Proposed the following empirical equations

$$\Theta_c(X_2 \le 0.1) = (0.047 \log X_2 - 0.023)/\beta_2 \qquad (2.40a)$$

$$\Theta_c(0.1 < X_2 \le 0.25) = (0.01 \log X_2 + 0.034)/\beta_2 \qquad (2.40b)$$

$$\Theta_c(X_2 > 0.25) = (0.0517 \log X_2 + 0.057)/\beta_2 \qquad (2.40c)$$

where $X_2 \approx 4.67 \times 10^{-3}[\Delta g/(\nu^2 \beta_2)]^{1/3} d$; $\beta_2 = (M + 2)/(1 + 2M)$; and M = uniformity coefficient of **Kramer** (1935) varying from 0.265 to 1

- **Iwagaki** (1956) considered the equilibrium of a single spherical particle, placed on a rough surface

$$\Theta_c = \frac{\cot \varphi}{\varepsilon_s \Psi_s R_*} \qquad (2.41)$$

where ε_s = empirical coefficient to take care of the sheltering effect; and Ψ_s = function of R_*.

- **Egiazaroff** (1965) presented yet another derivation for Θ_c as a function of R_*
- He assumed that at threshold condition, the velocity at an elevation of 0.63d (above the bottom of particle) equals the fall velocity w_{ss} of particle

$$\Theta_c = \frac{1.33}{C_D[a_r + 5.75\log(0.63)]} \qquad (2.42)$$

where a_r = 8.5; and C_D = drag coefficient = 0.4 for large R_*, and both a_r and C_D increase for low R_*

- **Mantz** (1977) proposed the extended Shields diagram
- **Yalin and Karahan** (1979) presented a graphical presentation of Θ_c versus R_*. It is regarded as a superior curve to the Shields Diagram

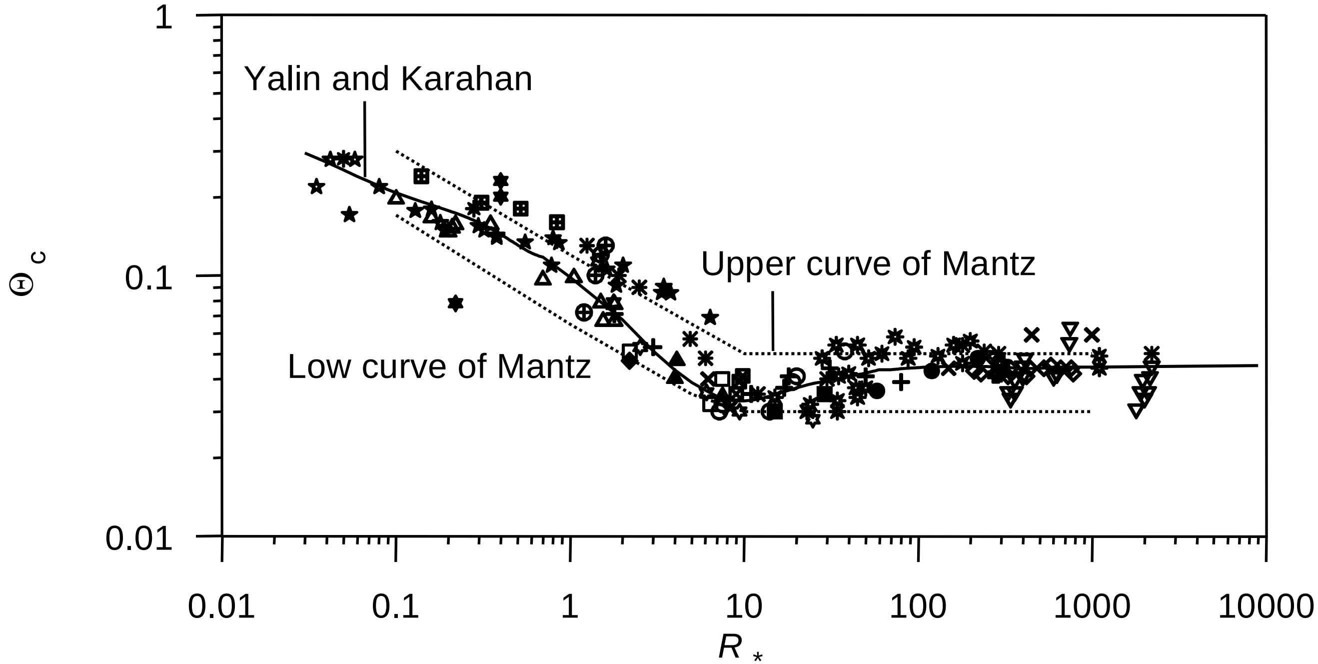


Fig. 2.6 Curves (Θ_c versus R_*) of **Mantz** and **Yalin and Karahan**

- **Soulsby and Whitehouse** (1997) presented the threshold in terms of the dimensionless particle size D_* and avoid the trial and error

$$\Theta_c = \frac{0.24}{D_*} + 0.055\,[1 - \exp(-0.2 D_*)] \qquad (2.43)$$

Probabilistic Concept

- The threshold of sediment motion is probabilistic in nature
- The concept gives the mean condition that there is a fifty percent chance for a given particle to move under specific flow and sediment conditions
- **Gessler** (1970) measured the probability that particles of a specific size stay
- It was shown that the probability of a given particle to stay depends strongly on the Shields parameter and weakly on particle Reynolds number

$$P_0(d) = \int_{d_{\min}}^{d_{\max}} p_0(d)\,dd \qquad (2.44)$$

where p_0 = frequency function of the original distribution

- The armor layer particle size frequency is

$$p_a(d) = k_1 q p_0(d) \qquad (2.45)$$

where q = probability for a particle size d to stay; and k_1 = constant

- The quantity q varies with particle size d that can be determined by

$$\int_{d_{\min}}^{d_{\max}} p_a(d)dd = 1 \qquad (2.46)$$

- The expression for particle size distribution of the armor layer is

$$P(d) = \frac{\int_{d_{\min}}^{d} qp_0(d)dd}{\int_{d_{\min}}^{d_{\max}} qp_0(d)dd} \qquad (2.47)$$

- The expression for particle size distribution of the moving particles

$$P(d) = \frac{\int_{d_{\min}}^{d} (1-q)p_0(d)dd}{\int_{d_{\min}}^{d_{\max}} (1-q)p_0(d)dd} \qquad (2.48)$$

- The most detailed experimental observations on the bed shear stress fluctuation carried out so far are due to **Grass** (1970)
- He pointed out that for any given area of a flat bed there is a random distribution of bed shear stresses due to stream flow
- There is a second independent random distribution of bed shear stress for the same area, at which the bed particles move

Dey's Sediment Threshold Model

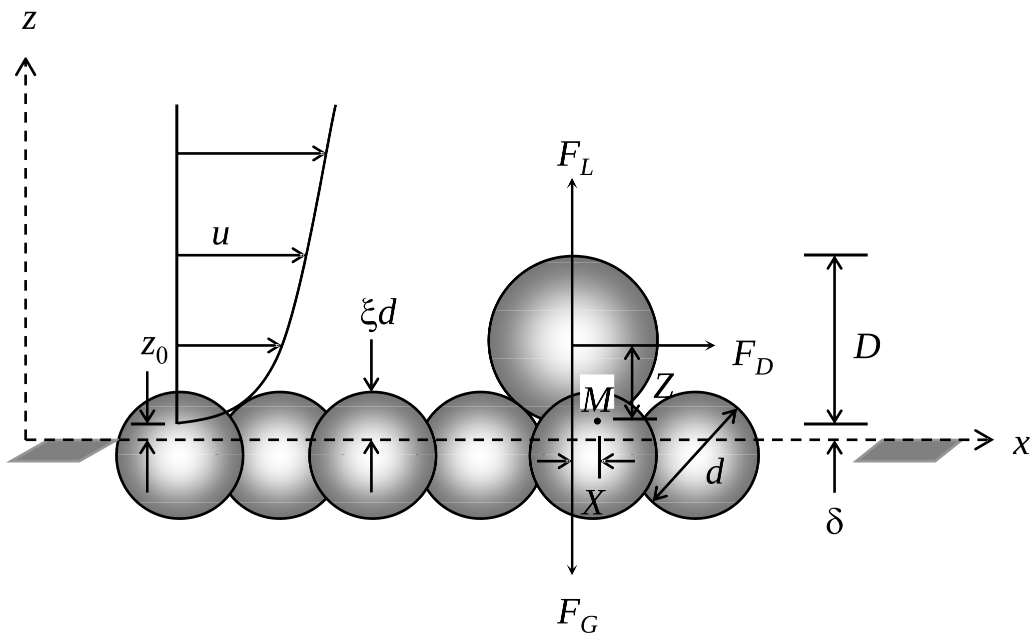


Fig. 2.7 Forces acting on a spherical solitary particle

Dey (1999) put forward a sediment threshold model

- Depending on the orientation of the bed particles, the solitary particle has a tendency either to roll over the valley formed by the two particles or to roll over the summit of a single particle
- The equation of moment about the point of contact *M* of the solitary particle downstream is

$$(F_L - F_G)X + F_D Z = 0 \qquad (2.49)$$

- The expressions of *X* and *Z* (horizontal and vertical lever arms) given by **Dey et al.** (1999) [also see **Dey** (1999)] are

$$X = \frac{\sqrt{3}}{4} \cdot \frac{Dd}{D+d} \qquad (2.50)$$

$$Z = \frac{1}{2\sqrt{3}} \cdot \frac{D}{D+d}(3D^2 + 6Dd - d^2)^{0.5} \qquad (2.51)$$

- The submerged weight of the solitary particle is

$$F_G = \frac{\pi}{6} D^3 (\rho_s - \rho) g \qquad (2.52)$$

- The drag force developed due to pressure and viscous skin frictional forces is

$$F_D = C_D \frac{\pi}{8} D^2 \rho u_m^2 \qquad (2.53)$$

where C_D = drag coefficient; and u_m = mean flow velocity received by the frontal area (the projected area of the particle being right angles to the direction of flow) of the solitary particle

- Empirical equation for drag coefficient C_D given by **Morsi and Alexander** (1972) is used

$$C_D = a + bR^{-1} + cR^{-2} \qquad (2.54)$$

where R = flow Reynolds number at particle level (= $u_m D/\nu$); and a, b and c = coefficients dependent on R

- The lift force, caused by the velocity gradient, in a shear flow is termed *lift due to shear effect* (F_{Ls})

- For a sphere in a viscous flow, **Saffman** (1968) proposed the following equation

$$F_{Ls} = C_L \rho D^2 u_m \left(\nu \frac{\partial u}{\partial z} \right)^{0.5} \qquad (2.55)$$

where $\partial u/\partial z$ = velocity gradient; u = time-averaged flow velocity at z

- For low particle Reynolds number R_*, Eq. (2.55) is applicable

- For large Reynolds number ($R_* > 3$), the solitary particle spins into the groove, formed by the three closely packed bed particles

- The lift force, caused by the spinning mode of particle, is termed *lift due to Magnus effect* (F_{Lm})
- **Rubinow and Keller** (1961) formulated it

$$F_{Lm} = C_L \rho D^3 u_m \omega \quad (2.56)$$

where ω = angular velocity of spinning particle

- According to **Saffman** (1965), the maximum angular velocity achieved by a solitary particle equals $0.5\partial u/\partial z$

$$F_{Lm} = 0.5 C_L \rho D^3 u_m \frac{\partial u}{\partial z} \quad (2.57)$$

- The total lift force F_L, a combination of F_{Ls}- and F_{Lm}, is expressed as

$$F_L = C_L \rho D^2 u_m \left(\frac{\partial u}{\partial z}\right)^{0.5} \left[\nu^{0.5} + 0.5 f(R_*)\, D \left(\frac{\partial u}{\partial z}\right)^{0.5} \right] \quad (2.58)$$

where $f(R_*) = 1$ for $R_* \geq 3$; $f(R_*) = 0$ for $R_* < 3$; and R_* is the particle Reynolds number ($= u_* d/\nu$). For low values of R_* ($R_* < 3$), particles do not spin

Using Eqs. (2.50) - (2.53) and (2.58) into Eq. (2.49), the equation for the threshold of sediment motion is obtained

$$\Theta_c = \frac{2\pi\hat{d}}{\pi C_D \hat{u}_m^2 (3 + 6\hat{d} - \hat{d}^2)^{0.5} + 6 C_L \hat{d}\hat{u}_m (\partial\hat{u}/\partial\hat{z})\{2[(R_*/\hat{d})\partial\hat{u}/\partial\hat{z}]^{-0.5} + f(R_*)\}} \tag{2.59}$$

where $\hat{u}_m = u_m/u_*$; $\hat{d} = d/D$; $\hat{u} = u/u_*$; and $\hat{z} = z/D$

- The accuracy of the results obtained from the model is highly dependent on the accurate determination of $\hat{d}$
- To avoid this difficulty, $\hat{d}$ is determined from the information on angle of repose of bed sediments, using the expression given by **Ippen and Eagleson** (1955) for the spherical sediments

$$\hat{d} = \frac{2\tan\varphi\,[6\tan\varphi + (48\tan^2\varphi + 27)^{0.5}]}{4\tan^2\varphi + 9} \tag{2.60}$$

where φ = angle of repose

- The most important event for the threshold of sediment motion is the sweep event, which has a dominant role in entraining sediment particles at the bed

- The sweep event applies shear in the direction of the flow and provides additional forces to the viscous shear stress

- **Keshavarzy and Ball** (1996) reported that the magnitude of instantaneous bed shear stress in a sweep event is much larger than time-averaged bed shear stress. Thus, they proposed the following equation for rough-turbulent regime

$$u_{*t} = (1 + p\sqrt{\alpha - 1}\cos\psi)\, u_{*} = \eta_t u_{*} \qquad (2.61)$$

where u_{*t} = total shear velocity (= u_* + u_t); u_t = instantaneous shear velocity [= $u_* p(\alpha - 1)^{0.5}\cos\psi$ or $(\tau_t/\rho)^{0.5}$]; τ_t = instantaneous bed shear stress; p = probability of occurring sweep event; α is τ_t/τ_0 and ψ is the sweep angle

- Θ_c calculated from Eq. (2.59) is modified as

$$\Theta_c = \Theta_c(\text{Eq. } 2.59)/\eta_t^2 \qquad (2.62)$$

- The particle parameter $\tilde{d}$ is given by $(d/\nu)[gd(\rho_s-\rho)/\rho]^{0.5}$. The following equation is used to compute $\tilde{d}$

$$\tilde{d} = R_*(\hat{d}/\Theta_c)^{0.5} \quad (2.63)$$

- The virtual bed level is considered to be at a depth of ξd below the top of the bed particles

- The normal distance δ between the virtual bed level and the bottom level of the solitary sediment particle given by **Dey et al.** (1999) is

$$\delta = \frac{1}{2\sqrt{3}}(3D^2 + 6Dd - d^2)^{0.5} - \frac{1}{2}(D+d) + \xi d \quad (2.64)$$

- The mean velocity of flow received by the frontal area of the solitary particle is

$$u_m = \frac{2\zeta}{A}\int_{\varepsilon}^{D+\delta} u[(z-\delta)(D+\delta-z)]^{0.5} dz \quad (2.65)$$

where A = frontal area of the solitary particle exposed to the flow, that is $(\pi D^2/4)\{1-\arccos(1-2\hat{h}) + 2(1-2\hat{h})[(1-\hat{h})]^{0.5}\}$; $\hat{h} = h/D$; $h = \varepsilon - \delta$; and ζ = coefficient (< 1) and ε = normal distance between the bottom level of the solitary particle or zero-velocity level and the virtual bed level

- The introduction of ζ is pertinent here because the summits of the bed particles upstream of the solitary particle obstruct the velocity of flow to some extent
- The normalized mean velocity $\hat{u}_m$ is obtained

$$\hat{u}_m = \frac{2\zeta}{\hat{A}} \int_{\hat{\varepsilon}}^{1+\hat{\delta}} \hat{u}[(\hat{z}-\hat{\delta})(1+\hat{\delta}-\hat{z})]^{0.5} d\tilde{z} \quad (2.66)$$

where $\hat{A} = A/D^2$; $\hat{\delta} = \delta/D$; and $\hat{\varepsilon} = \varepsilon/D$

- The velocity gradient $\partial u/\partial z$ can be obtained

$$\frac{\partial u}{\partial z} = \frac{1}{D+\delta-\varepsilon} \int_{\varepsilon}^{D+\delta} \frac{\partial u}{\partial z} dz = \frac{u_{D+\delta} - u_{\varepsilon}}{D+\delta-\varepsilon} \quad (2.67)$$

- The normalized velocity gradient $\partial\hat{u}/\partial\hat{z}$ is

$$\frac{\partial \hat{u}}{\partial \hat{z}} = \frac{\hat{u}_{1+\hat{\delta}} - \hat{u}_{\hat{\varepsilon}}}{1+\hat{\delta}-\hat{\varepsilon}} \quad (2.68)$$

Case 1 ($R_* < 3$):

- The flow is hydraulically smooth when R_* is less than three because the bed roughness lies within the viscous sub-layer
- It is assumed that the velocity distribution of the flow is solely linear for $R_* < 3$

$$\hat{u} = \frac{z u_*}{\nu} \qquad (2.69)$$

- Mean velocity $\hat{u}_m$ obtained

$$\hat{u}_m = \frac{2\zeta R_*}{\hat{A}\hat{d}} \int_{\hat{\varepsilon}}^{1+\hat{\delta}} [(\hat{z} - \hat{\delta})(1 + \hat{\delta} - \hat{z})]^{0.5} \hat{z} d\hat{z} \qquad (2.70)$$

where $\hat{\varepsilon} = 0$ if $\hat{\delta} \leq 0$ and $\hat{\varepsilon} = \hat{\delta}$ if $\hat{\delta} > 0$

- The velocity gradient determined using Eq. (2.70)

$$\frac{\partial \hat{u}}{\partial \hat{z}} = \frac{R_*}{\hat{d}} \qquad (2.71)$$

Case 2 ($3 \le R_* \le 70$):

- The range of particle Reynolds number $3 \le R_* \le 70$ can be considered as transitional regime
- The equation of the velocity distribution for transitional regime proposed by **Reichardt** (1951)

$$\hat{u} = \frac{1}{\kappa}\left\{\ln\left(1+\frac{\kappa \hat{z} R_*}{\hat{d}}\right) - \left[1-\exp\left(-\frac{\hat{z}R_*}{11.6\hat{d}}\right) - \frac{\hat{z}R_*}{11.6\hat{d}}\exp\left(-\frac{\hat{z}R_*}{3\hat{d}}\right)\right]\ln\left(\frac{\kappa \hat{z}_0 R_*}{\hat{d}}\right)\right\} \quad (2.72)$$

where κ = von Karman constant (= 0.4); z_0 = zero-velocity level above the virtual bed level (= $0.033k_s$); and k_s = equivalent roughness height of Nikuradse, assumed as d (**Wiberg and Smith** 1987)

- Mean velocity $\hat{u}_m$ obtained

$$\hat{u}_m = \frac{2\zeta}{\kappa \hat{A}}\int_{\hat{\varepsilon}}^{1+\hat{\delta}} [(\hat{z}-\hat{\delta})(1+\hat{\delta}-\hat{z})]^{0.5}\left\{\ln\left(1+\frac{\kappa \hat{z} R_*}{\hat{d}}\right)\right.$$

$$\left. - \left[1-\exp\left(-\frac{\hat{z}R_*}{11.6\hat{d}}\right) - \frac{\hat{z}R_*}{11.6\hat{d}}\exp\left(-\frac{\hat{z}R_*}{3\hat{d}}\right)\right]\ln\left(\frac{\kappa \hat{z}_0 R_*}{\hat{d}}\right)\right\} d\hat{z} \quad (2.73)$$

where $\hat{\varepsilon} = \hat{z}_0$ if $(\hat{z}_0 - \hat{\delta}) \ge 0$ and $\hat{\varepsilon} = \hat{\delta}$ if $(\hat{z}_0 - \hat{\delta}) < 0$

- The velocity gradient obtained using Eq. (2.72)

$$\frac{\partial \hat{u}}{\partial \hat{z}} = \frac{1}{\kappa(1+\hat{\delta}-\hat{\varepsilon})}\left\{\ln\left[1+\frac{\kappa(1+\hat{\delta})R_*}{\hat{d}}\right] - \ln\left(1+\frac{\kappa\hat{\varepsilon}R_*}{\hat{d}}\right)\right\} + \frac{1}{\kappa(1+\hat{\delta}-\hat{\varepsilon})}\left\{\exp\left[-\frac{(1+\hat{\delta})R_*}{11.6\hat{d}}\right]\right.$$

$$\left. - \exp\left(-\frac{\hat{\varepsilon}R_*}{11.6\hat{d}}\right) + \frac{(1+\hat{\delta})R_*}{11.6\hat{d}}\exp\left[-\frac{(1+\hat{\delta})R_*}{3\hat{d}}\right] - \left(\frac{\hat{\varepsilon}R_*}{11.6\hat{d}}\right)\exp\left(-\frac{\hat{\varepsilon}R_*}{3\hat{d}}\right)\right\}\ln\left(\frac{\kappa\hat{z}_0 R_*}{\hat{d}}\right)$$

(2.74)

Case 3 (R_* > 70):

- The velocity distribution in rough regime is

$$\hat{u} = \frac{1}{\kappa}\ln\left(\frac{z}{z_0}\right) \quad (2.75)$$

- Mean velocity $\hat{u}_m$ obtained

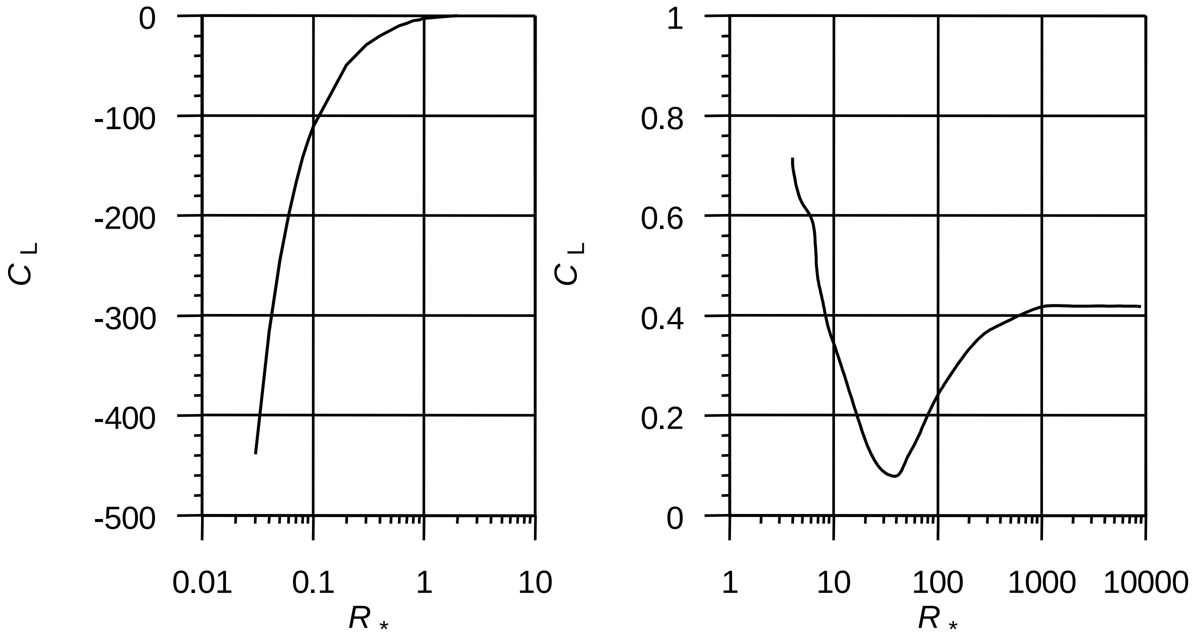
$$\hat{u}_m = \frac{2\zeta}{\kappa\hat{A}}\int_{\hat{\varepsilon}}^{1+\hat{\delta}}[(\hat{z}-\hat{\delta})(1+\hat{\delta}-\hat{z})]^{0.5}\ln\left(\frac{\hat{z}}{\hat{z}_0}\right)d\hat{z} \quad (2.76)$$

- The velocity gradient can be determined

$$\frac{\partial \hat{u}}{\partial \hat{z}} = \frac{1}{\kappa(1+\hat{\delta}-\hat{\varepsilon})}\ln\left(\frac{1+\hat{\delta}}{\hat{\varepsilon}}\right) \qquad (2.77)$$

- Simpson's rule can be applied to solve Eqs. (2.70), (2.73) and (2.76)

- As the exact expression for the lift coefficient C_L as a function of R_* is not available, Eq. (2.62) is required to be calibrated extensively



- The negative values of C_L for low range of R_* (R_* < 3) were also reported by **Watters and Rao** (1971) and **Davies and Samad** (1978)

Fig. 2.8 Dependency of C_L on R_*

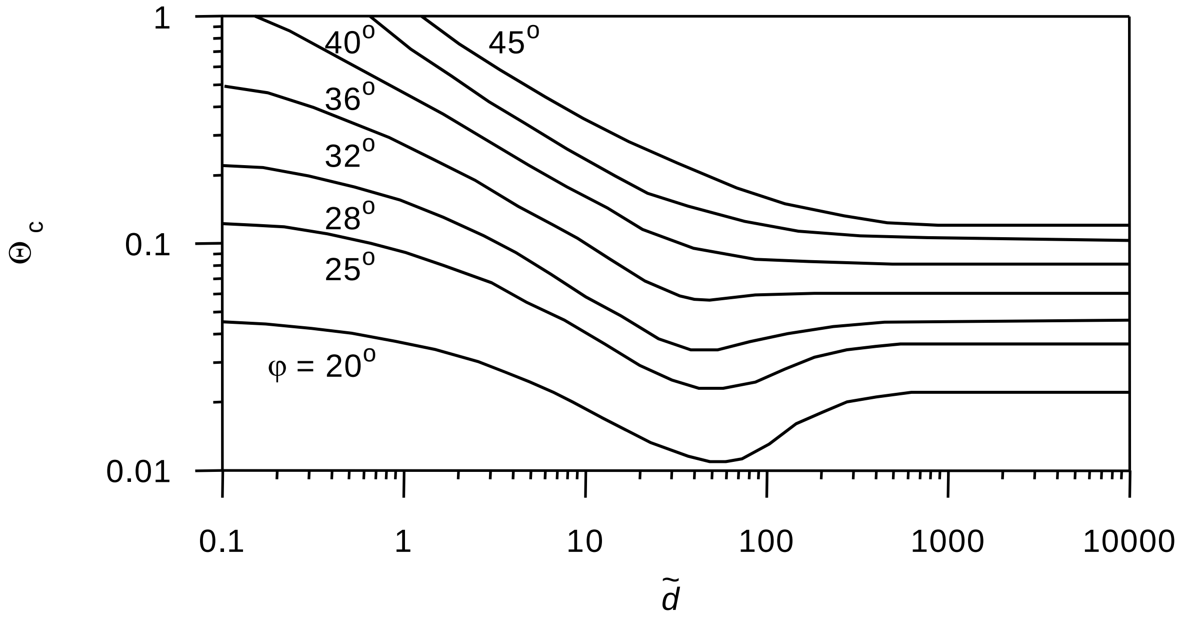


Fig. 2.9 Dependency of Θ_c on particle parameter $\tilde{d}$ for different φ

- Fig. 2.9 enables direct estimation of Θ_c

Sediment Threshold on Arbitrary Sloping Beds: Dey's Approach

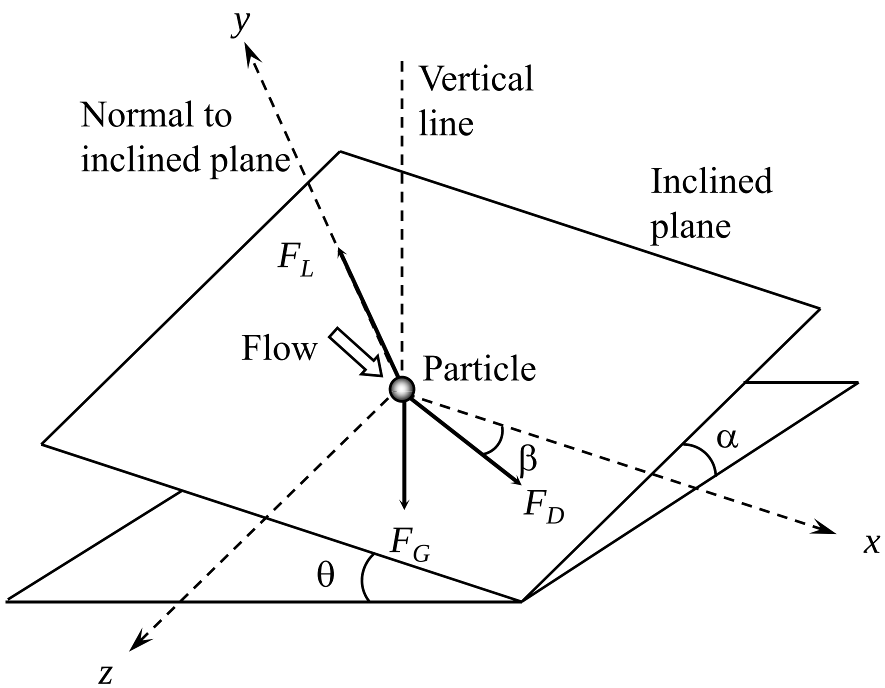


Fig. 2.10 Forces acting on a particle lying on an arbitrary sloping bed

Dey (2003) gave a sediment threshold model for arbitrary sloping beds

- When the solitary particle is about to move downstream from its original position, the equation of force balance

$$F_s^2 = (F_D \cos\beta + F_G \sin\theta)^2 + (F_D \sin\beta + F_G \sin\alpha)^2 \quad (2.78)$$

where F_s = static Coulomb friction force between particle and bed; θ = longitudinal bed angle with horizontal; α = transverse bed angle with horizontal; and β = angle of inclination of flow with respect to the longitudinal axis of the channel (positive downward)

- The submerged weight of the particle

$$F_G = \frac{\pi}{6} d^3 (\rho_s - \rho) g \qquad (2.79)$$

- The static Coulomb friction force is equated to

$$F_s = (F_G \sqrt{\cos^2 \theta - \sin^2 \alpha} - F_L) \mu_c \qquad (2.80)$$

where μ_c = static Coulomb friction factor at threshold condition

Equating Eqs. (2.78) and (2.80)

$$F_D^2 + 2F_G(\cos\beta \sin\theta + \sin\beta \sin\alpha) + F_G^2(\sin^2\theta + \sin^2\alpha)$$

$$-(F_G \sqrt{\cos^2\theta - \sin^2\alpha} - F_L)^2 \tan^2\varphi = 0 \qquad (2.81)$$

Normalizing the above equation

$$(1-\eta^2\tan^2\varphi)\Theta_{cs}^2+\frac{2}{\hat{F}_D}(\cos\beta\sin\theta+\sin\beta\sin\alpha+\eta\tan^2\varphi\sqrt{\cos^2\theta-\sin^2\alpha}\,\hat{\Theta}_{cs}$$

$$-\frac{1}{\hat{F}_D}[(\cos^2\theta-\sin^2\alpha)\tan^2\varphi-\sin^2\theta-\sin^2\alpha]=0 \qquad (2.82)$$

where $\eta = F_L/F_D$; Θ_{cs} = Shields parameter on an arbitrarily sloping bed, that is $\rho u_{*s}{}^2/[(\rho_s-\rho)gd]$ or $\tau_{0bs}/[(\rho_s-\rho)gd]$; u_{*s} = critical shear velocity on a sloping bed, that is $(\tau_{0s}/\rho)^{0.5}$; τ_{0s} = critical bed shear stress on a sloping bed; and $\hat{F}_D = 6F_D/(\pi\rho d^2 u_{*s}{}^2)$

- The value of η proposed by **Chepil** (1958) is as 0.85
- The positive solution of Eq. (2.82)

$$\Theta_{cs}=\frac{1}{(1-\eta^2\tan^2\varphi)\hat{F}_D}\{-(\cos\beta\sin\theta+\sin\beta\sin\alpha+\eta\tan^2\varphi\sqrt{\cos^2\theta-\sin^2\alpha})$$

$$+[(\cos\beta\sin\theta+\sin\beta\sin\alpha+\eta\tan^2\varphi\sqrt{\cos^2\theta-\sin^2\alpha})^2$$

$$+(1-\eta^2\tan^2\varphi)(\cos^2\theta\tan^2\varphi-\sin^2\alpha\tan^2\varphi-\sin^2\theta-\sin^2\alpha)]^{0.5}\} \qquad (2.83)$$

- For a horizontal bed, θ and α become zero and Eq. (2.83) reduces

$$\hat{\Theta}_c = \frac{\tan\varphi}{(1+\eta\tan\varphi)\hat{F}_D} \tag{2.84}$$

Dividing Eq. (2.83) by Eq. (2.84), yields

$$\tilde{\Theta}_{cs} = \frac{1}{(1-\eta\tan\varphi)\tan\varphi}\{-(\cos\beta\sin\theta+\sin\beta\sin\alpha+\eta\tan^2\varphi\sqrt{\cos^2\theta-\sin^2\alpha})$$
$$+[(\cos\beta\sin\theta+\sin\beta\sin\alpha+\eta\tan^2\varphi\sqrt{\cos^2\theta-\sin^2\alpha})^2$$
$$+(1-\eta^2\tan^2\varphi)(\cos^2\theta\tan^2\varphi-\sin^2\alpha\tan^2\varphi-\sin^2\theta-\sin^2\alpha)]^{0.5}\} \tag{2.85}$$

where $\tilde{\Theta}_{cs}$= critical bed shear stress ratio, that is τ_{0s}/τ_0

- The flow through a river or channel is in the longitudinal direction

- The equation of $\widetilde{\Theta}_{cs}$ for this type of flow can be obtained using $\beta = 0$

$$\widetilde{\Theta}_{cs} = \frac{1}{(1-\eta\tan\varphi)\tan\varphi}\{-(\sin\theta + \eta\tan^2\varphi\sqrt{\cos^2\theta - \sin^2\alpha})$$

$$+[(\sin\theta + \eta\tan^2\varphi\sqrt{\cos^2\theta - \sin^2\alpha})^2$$

$$+(1-\eta^2\tan^2\varphi)(\cos^2\theta\tan^2\varphi - \sin^2\alpha\tan^2\varphi - \sin^2\theta - \sin^2\alpha)]^{0.5}\} \quad (2.86)$$

- For transverse bed slopes, using $\theta = 0$ and $\eta = 0$

$$\widetilde{\Theta}_{c\alpha} = \cos\alpha\sqrt{1 - \frac{\tan^2\alpha}{\tan^2\varphi}} \quad (2.87)$$

where $\widetilde{\Theta}_{c\alpha} = \tau_{0\alpha}/\tau_0$; and $\tau_{0\alpha}$ = bed shear stress on a transversely sloping bed

- For longitudinal bed slopes, using α = 0, Eq. (2.86) becomes

$$\tilde{\Theta}_{c\theta} = \cos\theta\left(1 - \frac{\tan\theta}{\tan\varphi}\right) \quad (2.88)$$

where $\tilde{\Theta}_{c\theta} = \tau_{0\theta}/\tau_0$; and $\tau_{0\theta}$ = bed shear stress on a longitudinally sloping bed

- **van Rijn** (1993) and **Dey** (2004) proposed that critical bed shear stress on an arbitrary sloping bed is given by $\tau_{0b} = \tau_0 \tilde{\Theta}_{c\alpha} \tilde{\Theta}_{c\theta}$

Streambed Armoring

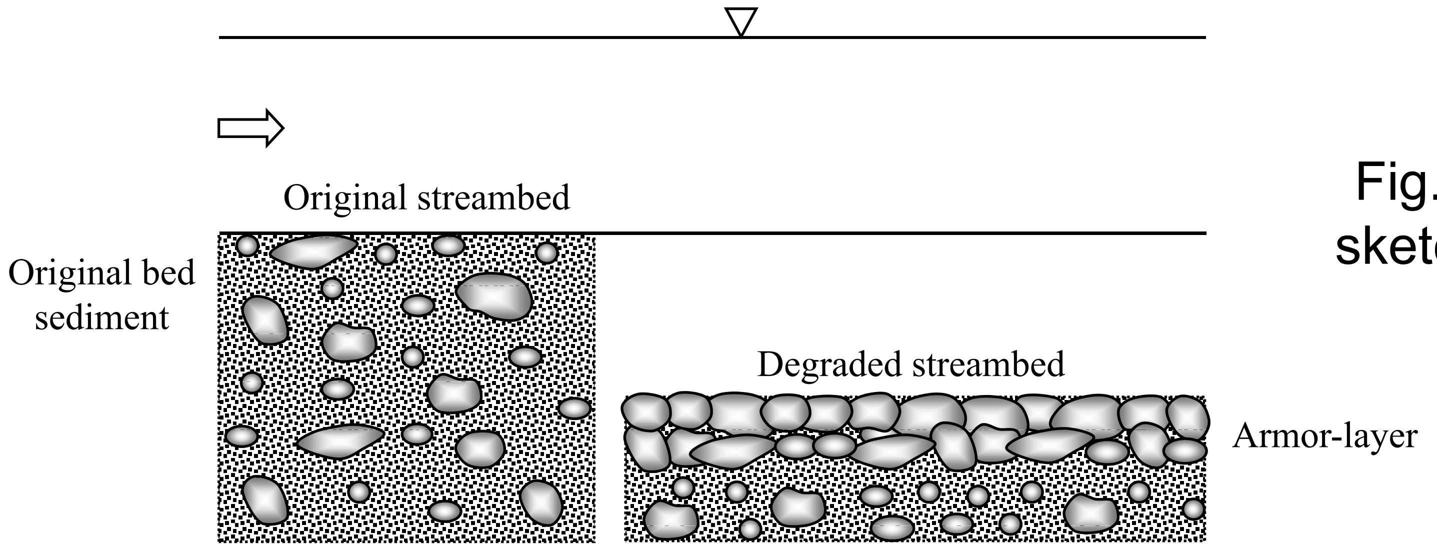


Fig. 2.11 Definition sketch of streambed armoring

In nonuniform sediments, finer sediment is transported at a faster rate than the coarser sediment, and the remaining bed sediment becomes coarser. This coarsening process stops until a layer of coarse sediment completely develops to cover the streambed protecting the finer sediments beneath it from being transported. Once this process is completed, the streambed is *armored* and the coarse layer is called the *armor-layer.* **Borah** (1989) and **Froehlich** (1995) reported that the natural armor-layer thickness is one to three times the armoring particle sizes.

Thank You