FLOW CHARACTERISTICS

Subhasish Dey, *Chair Professor*

Department of Civil Engineering
Indian Institute of Technology
Kharagpur, West Bengal
INDIA

Types of Flow

Fluid Flow

Laminar Flow

Layers of fluid slide smoothly over each other with different flow velocities without microscopic mixing of fluid particles normal to the direction of flow

Turbulent Flow

Velocity and the pressure at a fixed point in space do not remain constant at time but fluctuate very irregularly with high frequency

Turbulent Flow:

- Convenient to describe the hydrodynamic quantities by separating the time-averaged values from their fluctuations
- Decomposition of an instantaneous value of a hydrodynamic quantity is called the *Reynolds decomposition*

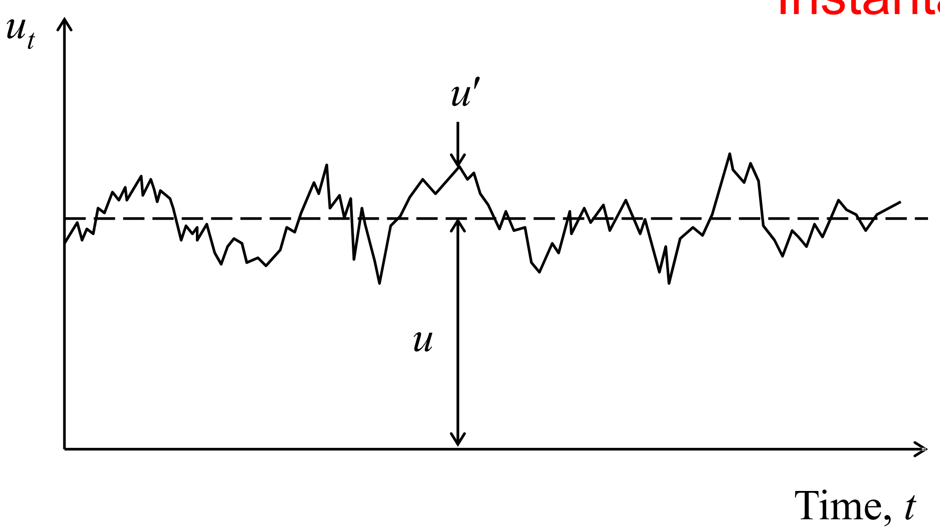


Fig. 1.1 Time series of u_t

Instantaneous velocity components

$$u_t = u + u' \quad (1.1a)$$

$$v_t = v + v' \quad (1.1b)$$

$$w_t = w + w' \quad (1.1c)$$

Pressure intensity

$$p_t = p + p' \quad (1.1d)$$

Time-averaged value of a hydrodynamic quantity, *u*

$$u = \frac{1}{t_1} \int_{t_0}^{t_0 + t_1} u_t \, dt \qquad (1.2)$$

where t_0 = any arbitrary time; t_1 = time over which the mean is taken

- t_1 is taken as sufficiently long interval of time in order to obtain the time independent quantities
- Time-averaged values of all the fluctuations are equal to zero

$$\overline{u'} = \overline{v'} = \overline{w'} = \overline{p'} = 0 \qquad (1.3a)$$

- Time-averaged values of the derivatives of velocity fluctuations also vanish

$$\overline{\frac{\partial u'}{\partial x}} = \overline{\frac{\partial^2 u'}{\partial x^2}} = \overline{\frac{\partial u u'}{\partial x^2}} = \cdots\cdots = 0 \qquad (1.3b)$$

- Quadratic terms resulting from the products of cross-velocity fluctuations such as $\overline{u'u'}$, $\overline{u'v'}$ and $\partial \overline{u'v'} / \partial x$ do not reduce to zero

Velocity fluctuations (u', v', w') influence the time-averaged velocity components (u, v, w), so that the (u, v, w) exhibits an apparent increase in the resistance to deformation, which is called as *turbulent stresses* or *Reynolds stresses*

Reynolds conditions written with two quantities E and G

$$\overline{E+G} = \overline{E} + \overline{G} \qquad (1.4a)$$

$$\overline{\text{constant} \times E} = \text{constant} \times \overline{E} \qquad (1.4b)$$

$$\overline{\text{constant}} = \text{constant} \qquad (1.4c)$$

$$\overline{\frac{\partial E}{\partial s_1}} = \frac{\partial \overline{E}}{\partial s_1} \qquad (1.4d)$$

$$\overline{\overline{E} \cdot G} = \overline{E} \cdot \overline{G} \qquad (1.4e)$$

$$\overline{\overline{E}} = \overline{E} \qquad (1.4f)$$

$$\overline{E'} = 0 \qquad (1.4g)$$

$$\overline{\overline{E} \cdot \overline{G}} = \overline{E} \cdot \overline{G} \qquad (1.4h)$$

$$\overline{\overline{E} \cdot G'} = \overline{E} \cdot \overline{G'} = 0 \qquad (1.4i)$$

where s_1 = space and time coordinate, that is (x, y, z, t)

Reynolds Equations and Reynolds Stresses

Navier-Stokes equations

$$\frac{\partial u_t}{\partial t}+u_t\frac{\partial u_t}{\partial x}+v_t\frac{\partial u_t}{\partial y}+w_t\frac{\partial u_t}{\partial z}=g_x-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u_t}{\partial x^2}+\frac{\partial^2 u_t}{\partial y^2}+\frac{\partial^2 u_t}{\partial z^2}\right) \quad (1.5a)$$

$$\frac{\partial v_t}{\partial t}+u_t\frac{\partial v_t}{\partial x}+v_t\frac{\partial v_t}{\partial y}+w_t\frac{\partial v_t}{\partial z}=g_y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v_t}{\partial x^2}+\frac{\partial^2 v_t}{\partial y^2}+\frac{\partial^2 v_t}{\partial z^2}\right) \quad (1.5b)$$

$$\frac{\partial w_t}{\partial t}+u_t\frac{\partial w_t}{\partial x}+v_t\frac{\partial w_t}{\partial y}+w_t\frac{\partial w_t}{\partial z}=g_z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w_t}{\partial x^2}+\frac{\partial^2 w_t}{\partial y^2}+\frac{\partial^2 w_t}{\partial z^2}\right) \quad (1.5c)$$

Continuity equation

$$\frac{\partial u_t}{\partial x}+\frac{\partial v_t}{\partial y}+\frac{\partial w_t}{\partial z}=0 \quad (1.5d)$$

where g_x, g_y, g_z = components of gravity in (x, y, z); ρ = mass density of fluid; and ν = kinematic viscosity of fluid

Eqs. (1.5a) - (1.5d) can be rewritten as

$$\frac{\partial u_t}{\partial t}+\frac{\partial (u_t^2)}{\partial x}+\frac{\partial (u_t v_t)}{\partial y}+\frac{\partial (u_t w_t)}{\partial z}=g_x-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u_t}{\partial x^2}+\frac{\partial^2 u_t}{\partial y^2}+\frac{\partial^2 u_t}{\partial z^2}\right) \quad (1.6a)$$

$$\frac{\partial v_t}{\partial t}+\frac{\partial (v_t u_t)}{\partial x}+\frac{\partial (v_t^2)}{\partial y}+\frac{\partial (v_t w_t)}{\partial z}=g_y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v_t}{\partial x^2}+\frac{\partial^2 v_t}{\partial y^2}+\frac{\partial^2 v_t}{\partial z^2}\right) \quad (1.6b)$$

$$\frac{\partial w_t}{\partial t}+\frac{\partial (w_t u_t)}{\partial x}+\frac{\partial (w_t v_t)}{\partial y}+\frac{\partial (w_t^2)}{\partial z}=g_z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\left(\frac{\partial^2 w_t}{\partial x^2}+\frac{\partial^2 w_t}{\partial y^2}+\frac{\partial^2 w_t}{\partial z^2}\right) \quad (1.6c)$$

$$\frac{\partial u_t}{\partial x}+\frac{\partial v_t}{\partial y}+\frac{\partial w_t}{\partial z}=0 \quad (1.6d)$$

Using the Reynolds condition and decomposition into Eqs. (1.6a) - (1.6d)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=g_x-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\nabla^2 u-\left(\frac{\partial \overline{u'u'}}{\partial x}+\frac{\partial \overline{u'v'}}{\partial y}+\frac{\partial \overline{u'w'}}{\partial z}\right) \quad (1.7a)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=g_y-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\nabla^2 v-\left(\frac{\partial \overline{v'u'}}{\partial x}+\frac{\partial \overline{v'v'}}{\partial y}+\frac{\partial \overline{v'w'}}{\partial z}\right) \quad (1.7b)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=g_z-\frac{1}{\rho}\frac{\partial p}{\partial z}+\nu\nabla^2 w-\left(\frac{\partial \overline{w'u'}}{\partial x}+\frac{\partial \overline{w'v'}}{\partial y}+\frac{\partial \overline{w'w'}}{\partial z}\right) \quad (1.7c)$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (1.7d)$$

where $\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

Subtracting Eq. (1.7d) from Eq. (1.6d), gives the continuity equation for fluctuating part

$$\frac{\partial u'}{\partial x}+\frac{\partial v'}{\partial y}+\frac{\partial w'}{\partial z}=0 \qquad (1.8)$$

- Last terms of Eqs. (1.7a) - (1.7c) are obtained from three cross products of velocity fluctuations and provide additional stresses developed due to turbulence
- They are called as turbulent shear stresses or *Reynolds stresses* and can be expressed as a stress tensor called *Reynolds stress tensor*

$$\begin{pmatrix} \sigma_u & \tau_{uv} & \tau_{uw} \\ \tau_{vu} & \sigma_v & \tau_{vw} \\ \tau_{wu} & \tau_{wv} & \sigma_w \end{pmatrix} = -\rho \begin{pmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'v'} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'w'} \end{pmatrix} \qquad (1.9)$$

where σ_u, σ_v, σ_w = normal stresses in (x, y, z) directions; and τ_{uv}, τ_{uw},τ_{vu}, τ_{vw}, τ_{wu}, τ_{wu} = shear stresses

- Reynolds stresses are developed due to turbulent fluctuations and are given by the time-averaged values of the quadratic terms in the turbulent fluctuations
- As these terms are added to the ordinary viscous stresses in the laminar flow and have a similar influence on the flow, it is called often *eddy viscosity*
- Reynolds stresses far outweigh the viscous stresses in turbulent flow

Reynolds Stress Distribution in Open Channel Flow

- For a steady flow having zero-pressure gradient in the *x*-direction (streamwise), the basic equations are continuity equation and two components of Reynolds equation

- *z*-component of Reynolds equation [Eq. (1.7c)] gives an equation for the Reynolds stress $-\rho\overline{w'w'}$

- *x*-component of Reynolds equation [Eq. (1.7a)] reduces to

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = g_x - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2}\right) - \left(\frac{\partial \overline{u'u'}}{\partial x} + \frac{\partial \overline{u'w'}}{\partial z}\right) \qquad (1.10)$$

- $\partial u/\partial t = 0$, $w = 0$, $g_x = -g\sin\theta$, $\partial p/\partial x = 0$ and u is a function of z only, where g = gravitational acceleration. Also, $\partial u/\partial x = 0$ for uniform flow

- Eq. (1.10) becomes

$$\mu\frac{d^2 u}{dz^2} + \frac{d(-\rho\overline{u'w'})}{dz} = \rho g\frac{dh}{dx} \qquad (1.11)$$

where μ = dynamic viscosity of fluid, that is $\rho\nu$

Eq. (1.11) can be written as

$$\frac{d}{dz}\left[\mu\frac{du}{dz}+(-\rho\overline{u'w'})\right]=\frac{\tau_0}{h} \qquad (1.12)$$

where τ_0 = bed shear stress, that is $\rho g h(dh/dx)$

- Terms inside the parenthesis in LHS of Eq. (1.12) are expressed as

$$\mu\frac{du}{dz}+-\rho\overline{u'w'}=\tau_v+\tau_l=\tau \qquad (1.13)$$

where τ_v = shear stress due to viscosity, τ_t = shear stress due to turbulence or Reynolds stress, and τ = total shear stress

- Eq. (1.12) becomes

$$d\tau/dz=\tau_0\,/\,h \qquad (1.14)$$

Integrating Eq. (1.14) yields

$$\tau = [1 - (z/h)]\tau_0 \qquad (1.15)$$

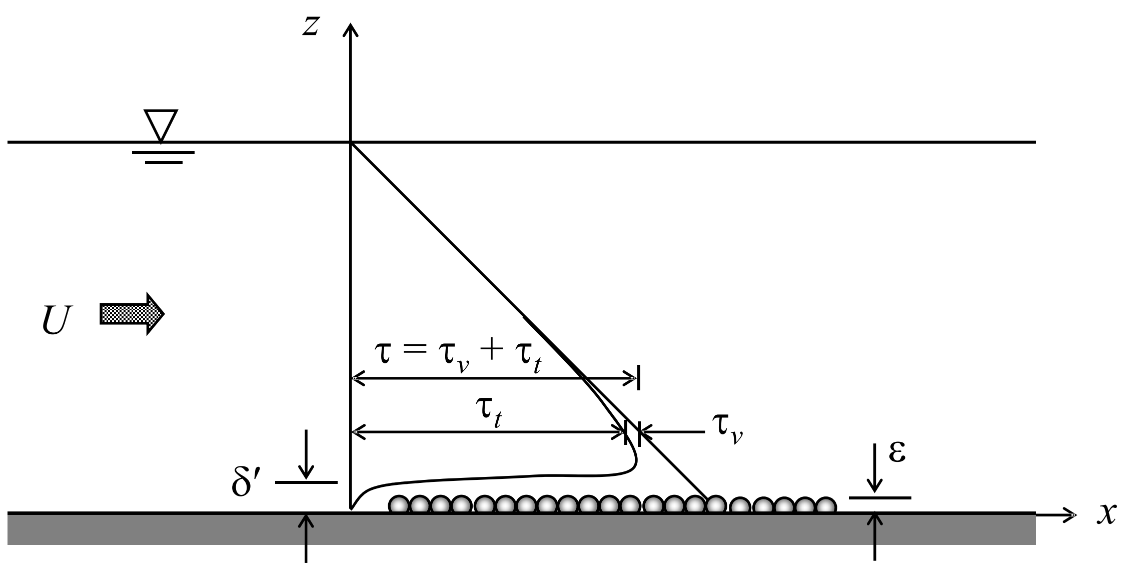


Fig. 1.2 Shear stress components and distribution

Classical Turbulence Theory

- No theory is available to describe the phenomena completely
- Existing theories are based on the semi-empirical hypothesis, which establish relationship between the Reynolds stresses caused in the exchange of momentum and the time-averaged velocities
- Basic theories proposed by **Prandtl** and **von Karman**

Prandtl's Mixing-Length Theory

- **Prandtl** introduced the mixing-length concept in order to calculate the turbulent shear stress or Reynolds stress

- He simulated momentum exchange on a macro-scale to that of the molecular motion of a gas to explain the mixing phenomenon induced by turbulence in fluid flow; and thus established the *mixing-length theory*

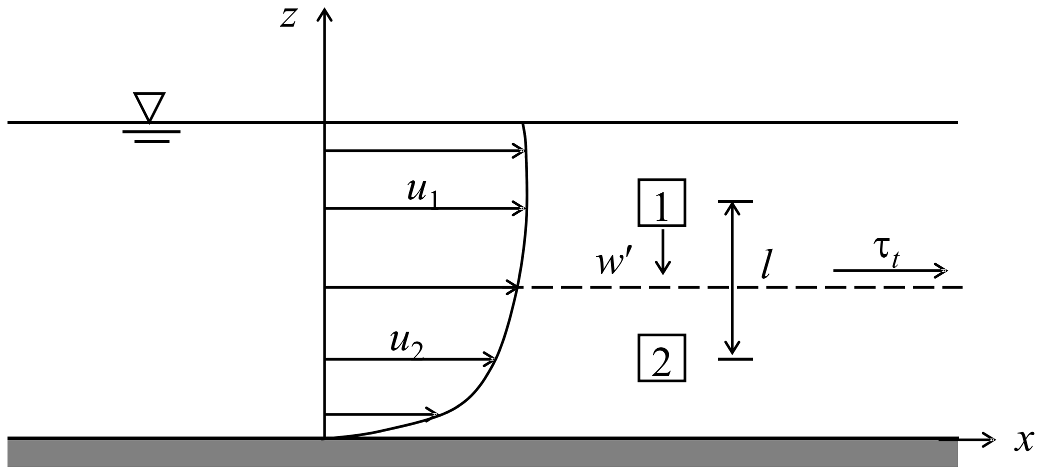


Fig. 1.3 Mixing-length in turbulent flow

- Horizontal instantaneous velocity fluctuation of the fluid element in layer 2 is

$$u' = u_2 - u_1 = l\frac{du}{dz} \qquad (1.16)$$

where *l* = mixing length

- Following the hypothesis of **Prandtl** that the vertical velocity fluctuation w' is of the same order of magnitude as u'

$$w' = -l \frac{du}{dz} \qquad (1.17)$$

- Turbulent shear stress or Reynolds stress τ_t becomes

$$\tau_t = -\rho(\overline{u'w'}) = \rho l^2 \left(\frac{du}{dz}\right)^2 \qquad (1.18)$$

- Yields the turbulent model of the mixing-length

$$\tau_t = \rho l^2 \left|\frac{du}{dz}\right| \left(\frac{du}{dz}\right) \qquad (1.19)$$

$$\tau_t = \rho \varepsilon_m \frac{du}{dz} \qquad (1.20)$$

where ε_m = kinematic eddy viscosity, that is $l^2(du/dz)$

- Using Eq. (1.20) into Eq. (1.13)

$$\tau = (\mu + \rho\varepsilon_m)\frac{du}{dz} = \rho(\nu + \varepsilon_m)\frac{du}{dz} \qquad (1.21)$$

According to Prandtl:

- Mixing-length *l* is proportional to the distance *z* from the boundary
- In flows along the smooth boundary, *l* must vanish, as the transverse motion is inhibited

$$l = \kappa z \qquad (1.22)$$

where κ = von Karman constant

Similarity Hypothesis of Turbulent Flow after von Karman

- Except in a region near the boundary, turbulence phenomena are not affected by the viscosity
- Basic pattern of turbulence at different positions are similar, i.e. they differ only in scales of time and length

$$l = \kappa \frac{du/dz}{d^2u/dz^2} \qquad (1.23)$$

- Eq. (1.23) indicates that the mixing-length is a local function and depends only on velocity distribution in the neighborhood of a particular point

Classification of Flow Layer

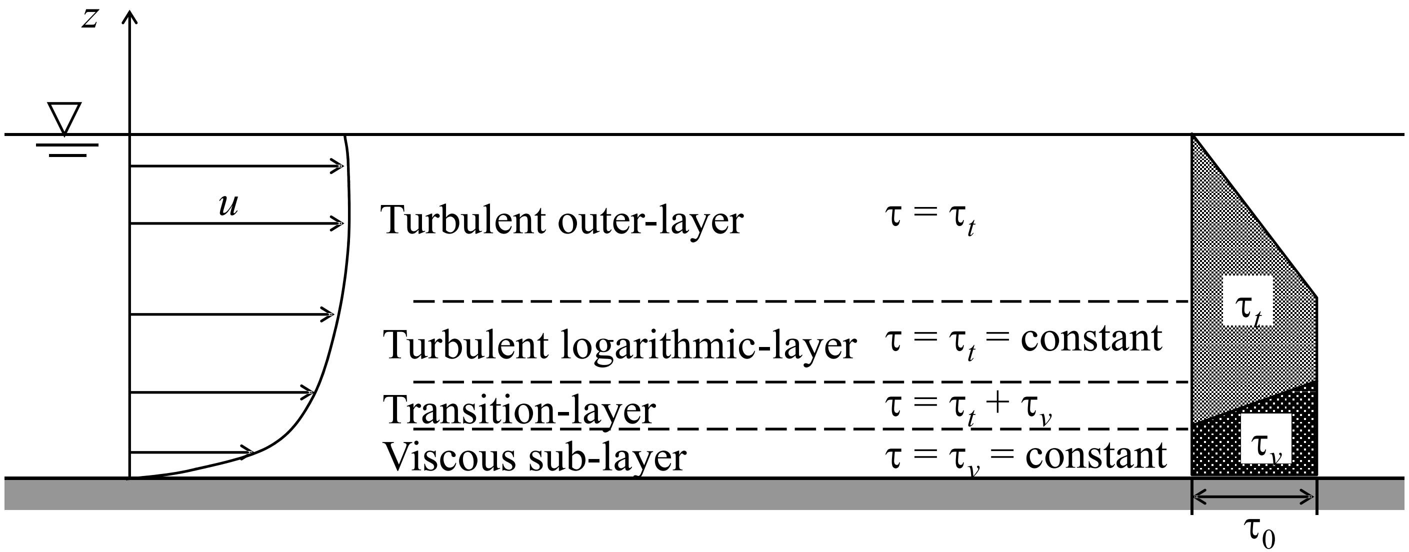


Fig. 1.4 Classification of flow region

- *Viscous sub-layer:* flow is laminar and turbulence is totally absent
- *Transition-layer or buffer-layer*: viscous and turbulence effects exist
- *Turbulent logarithmic-layer:* viscous shear stress is negligible and the shear stress is due to the turbulence only
- *Turbulent outer-layer:* velocities are almost constant because of the presence of large eddies, which produce strong mixing of flow

- Logarithmic velocity profile is applied to both the buffer- and turbulent outer-layers
- In the viscous sub-layer, the boundary roughness plays a role on the velocity distribution, which was first investigated by **Nikuradse** (1933)
- **Nikuradse** introduced the concept of equivalent roughness ε, called *Nikuradse's equivalent roughness*

Based on the experimental data, the flow is classified as

- **Hydraulically smooth flow** ($R_* \leq 5$): Bed roughness is smaller than the thickness of viscous sub-layer δ_v and not affect the velocity distribution
- **Hydraulically rough flow** ($R_* \geq 70$): Bed roughness is large that it produces eddies near the boundary and viscous sub-layer does not exist
- **Hydraulically transitional flow** ($5 < R_* < 70$): The velocity distribution is affected by both bed roughness and viscosity

where R_* = shear Reynolds number, that is $u_* \varepsilon/\nu$; and u_* = shear velocity, that is $(\tau_0/\rho)^{0.5}$

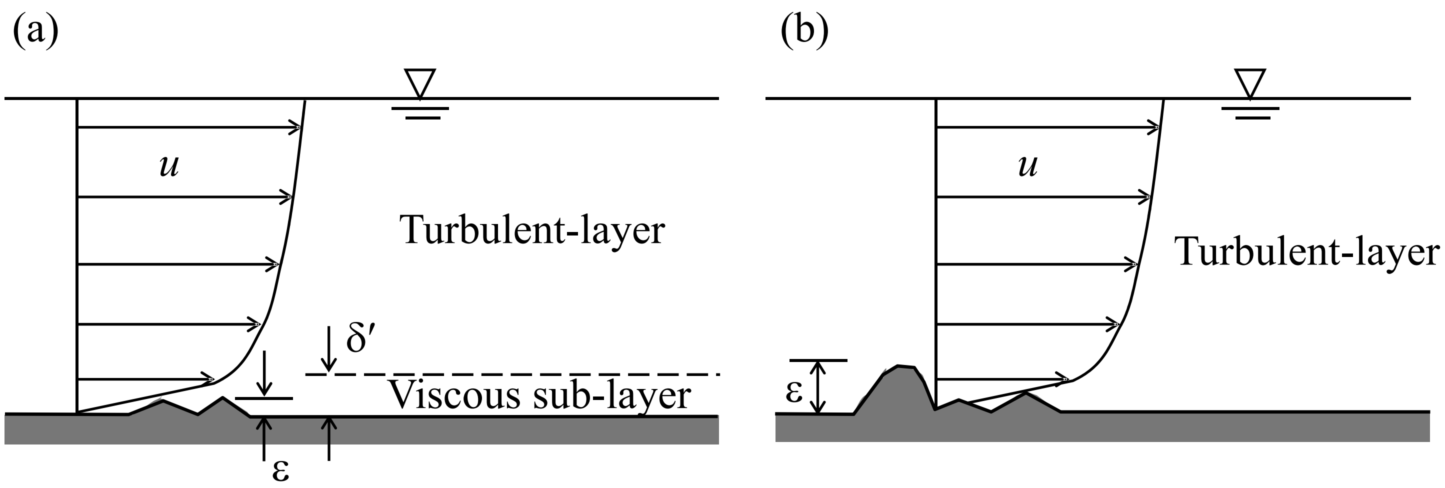


Fig. 1.5 (a) Hydraulically smooth flow and (b) hydraulically rough flow

Velocity Distributions

The flow zone over a boundary is characterized by the two-layer: an *inner-layer* where the turbulence is directly affected by the bed roughness and an *outer-layer* where the bed roughness indirectly influences the flow

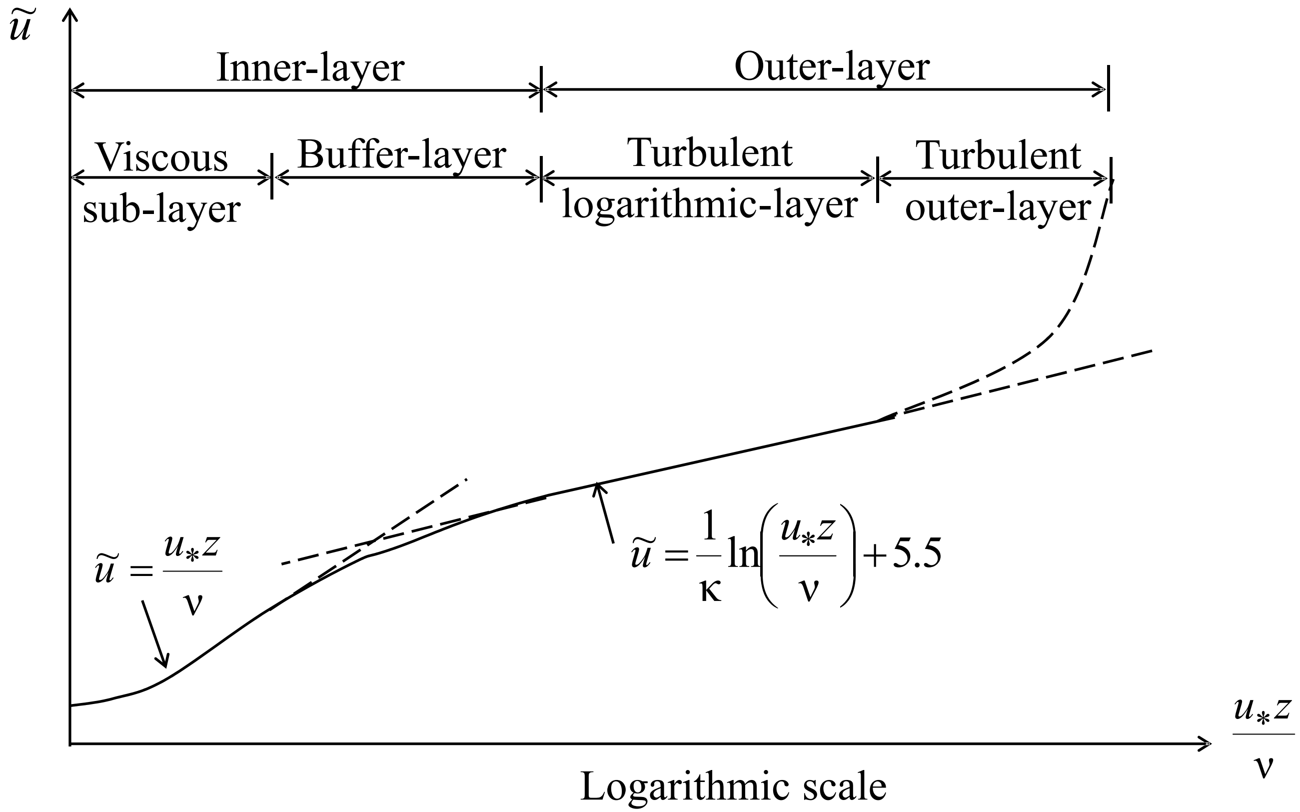


Fig. 1.6 Velocity profiles in different layers

Linear-Law in Viscous Sub-Layer

- In case of hydraulically smooth flow, the viscous shear stress is constant and equal to bed shear stress τ_0

$$\tau_v = \rho\nu \frac{du}{dz} = \tau_0 \qquad (1.24)$$

$$du = \frac{u_*^2}{\nu} dz \qquad (1.25)$$

- Integrating and using no-slip condition at the boundary, that is $u|_{z=0}$ = 0, yields

$$\tilde{u} = \frac{u_* z}{\nu} \qquad (1.26)$$

where $\tilde{u} = u/u_*$

- Linear velocity distribution in the viscous sub-layer
- Eq. (1.26) is valid for the range $0 < u_* z/\nu \leq 5$

Logarithmic-Law in Turbulent-Layer

- In the turbulent-layer, the total shear stress τ contains only the turbulent shear stress

$$\tau_t = \rho l^2 \left(\frac{du}{dz}\right)^2 = \tau_0 \qquad (1.27)$$

Putting $l = \kappa z$

$$du = \frac{u_*}{\kappa z} dz \qquad (1.28)$$

Integration of Eq. (1.28) gives the logarithmic velocity distribution

$$\tilde{u} = \frac{1}{\kappa} \ln z + \text{constant} \qquad (1.29)$$

Using the boundary condition $u = 0$ and $z = z_0$, that is zero-velocity level

$$\tilde{u} = \frac{1}{\kappa} \ln\left(\frac{z}{z_0}\right) \qquad (1.30)$$

According to **Nikuradse**'s study on pipe flows

$$z_0 = 0.11\frac{\nu}{u_*}$$ for smooth flow $R_* \leq 5$ (1.31a)

$$z_0 = 0.033\varepsilon$$ for rough flow $R_* \geq 70$ (1.31b)

$$z_0 = 0.11\frac{\nu}{u_*} + 0.033\varepsilon$$ for transition $5 < R_* < 70$ (1.31c)

- **American Society of Civil Engineers' Task Committee** (1963) reported that for open channel roughness similar to that encountered in pipes, the resistance equations similar to those of pipe flows are adequate

- For the flow over smooth boundary, such as a plane bed surface having median particle size less than 0.25 mm, using Eq. (1.31a) into Eq. (1.30)

$$\tilde{u} = \frac{1}{\kappa}\ln\left(\frac{u_* z}{\nu}\right) + 5.5 \qquad (1.32)$$

For the flow over a rough boundary, such as gravel-bed, which are often encountered in hilly rivers, Eq. (1.29) reduces to

$$\tilde{u} = \frac{1}{\kappa}\ln\tilde{z} + B_r \qquad (1.33)$$

where $\tilde{z} = z/\varepsilon$; B_r = constant of integration, that is $-(1/\kappa)\ln\tilde{z}_0$; and $\tilde{z}_0 = z_0/\varepsilon$

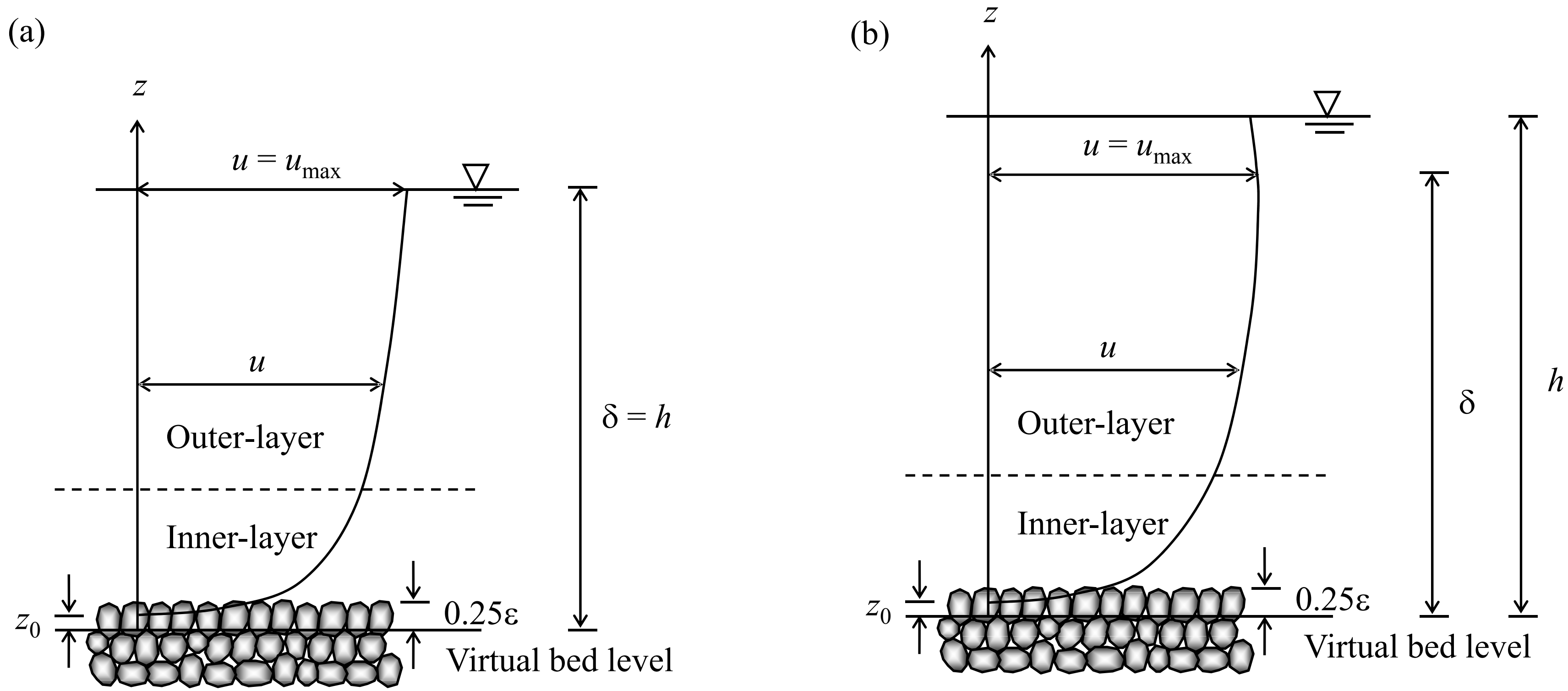
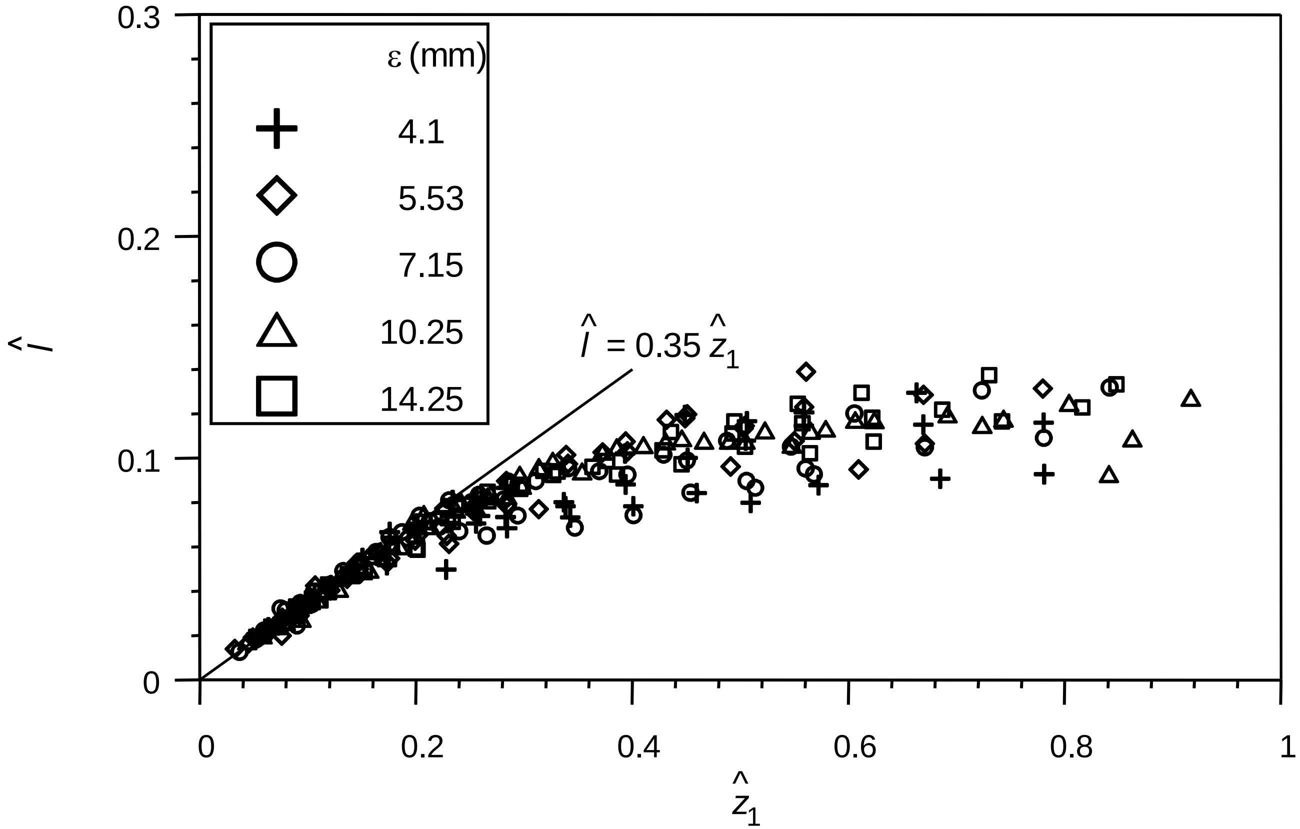


Fig. 1.7 Velocity profiles in gravel-beds: (a) wide channel and (b) deep channel

- **Nikora and Goring** (2000) reported that the value of κ goes down when the bed is mobile
- It is essential to estimate κ from the concept of the mixing-length using the experimental data
- **Raikar** (2006) used the measured velocity profiles to determine the velocity gradients *du/dz* by smooth curve fitting to the data
- He obtained the Reynolds stresses τ directly from the measured Reynolds stress distributions



- In the inner-layer, all the experimental data of the near-threshold condition collapse reasonably on a single band
- Mixing-length $\hat{l}$ increases linearly with increase in flow depth $\hat{z}_1$ up to $\hat{z}_1$ = 0.23, being in conformity with the Prandtl's hypothesis

Fig. 1.8 Mixing-length as a function of flow depth (**Raikar** 2006), where $\hat{z}_1 = z/\delta$

- Average value of the von Karman constant κ obtained from the slope of the fitted straight line is 0.35
- Value of κ is slightly less than that of **von Karman**'s 0.41 and greater than that of **Nikora and Goring**'s (2000) 0.29 for mobile gravel-beds
- Unrest condition of the surface particles at near-threshold is the principal cause of the reduction of the value of κ

- Upper-limit of the inner-layer obtained as $\hat{z}_1 = 0.23$, in this study, is slightly higher, since $\hat{z}_1 = 0.2$ is the traditional upper-limit (**Nezu and Nakagawa** 1993)
- Beyond $\hat{z}_1 = 0.23$, the mean trend of the curve is nonlinear becoming almost constant at $\hat{l} \approx 0.11$ towards the free surface
- Data trend of **Nezu and Rodi** (1986) and **Cardoso et al.** (1989) were similar as well towards the free surface
- In $\hat{z}_1 > 0.23$, the data shows a considerable scatter, which is also evident in **Nezu and Nakagawa** (1993), **Kironoto and Graf** (1994), and **Song et al.** (1994)
- Average value of B_r (and their standard deviation) obtained by **Raikar** (2006) for gravel-bed under near-threshold is 7.8 ($\pm$0.37)
- It is less than those reported in the literature for rough boundary streams

Law of Wake in Turbulent Outer-Layer (Coles Law)

- In the outer-layer the velocity profile deviates from the logarithmic-law, as the distance from the boundary increases ($u_* z/\nu > 1000$)

- Reason for this departure is owing to the assumption of constant shear stress throughout the fluid and mixing-length approximation

- **Coles** (1956) suggested the complete description of the velocity distribution u, including the law of the wake

$$\tilde{u} = \left[\frac{1}{\kappa}\ln\left(\frac{u_* z}{\nu}\right) + 5.5\right] + \frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}\hat{z}_1\right) \quad \text{for smooth flow} \qquad (1.34a)$$

$$\tilde{u} = \frac{1}{\kappa}\ln\tilde{z} + B_r + \frac{2\Pi}{\kappa}\sin^2\left(\frac{\pi}{2}\hat{z}_1\right) \quad \text{for rough flow} \qquad (1.34b)$$

where Π = Coles' wake parameter; and $\hat{z}_1 = z/\delta$

- Last term describes the velocity increase in the turbulent outer-layer and is called the *wake function*

- Wake function is zero near the boundary and increases gradually towards the free surface and reaches a maximum value of $2\Pi/\kappa$

- **Raikar** used the experimental velocity distributions to estimate the wake parameter Π from Eq. (1.34b) for gravel-bed under near-threshold
- The average value of Π calculated is 0.11 ($\pm$ 0.026)
- Due to the feeble movement of the surface particles at the near-threshold condition, the value of Π is slightly greater than those of fixed rough boundaries
- For smooth boundary streams, the values of Π are relatively high

Turbulence Characteristics in Flow over Loose Beds

- Turbulence fluctuations are presented in the form of root-mean-square (RMS) termed *turbulence intensity*

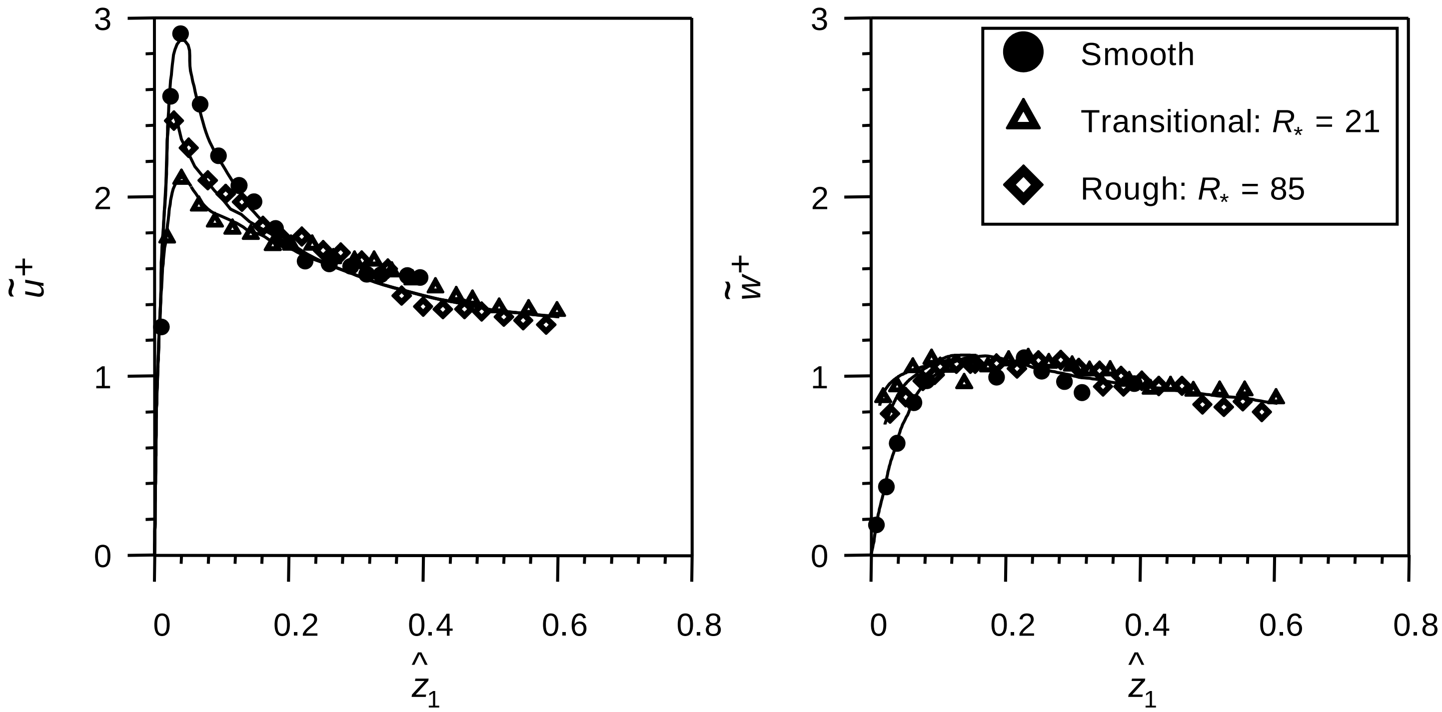


Fig. 1.9 Turbulence quantities (**Grass** 1971)

- Turbulence intensity is zero at a boundary and increases rapidly to reach its peak value within a slight distance from the boundary. Away from the boundary, in the main flow region, the turbulence intensity is less and essentially constant
- In the main flow region, the vertical fluctuations approach shear velocity, $w^+/u_* \approx 1$. The streamwise fluctuations are greater than the shear velocity
- Type of boundary has no effect on the intensity of the fluctuations in the main flow region

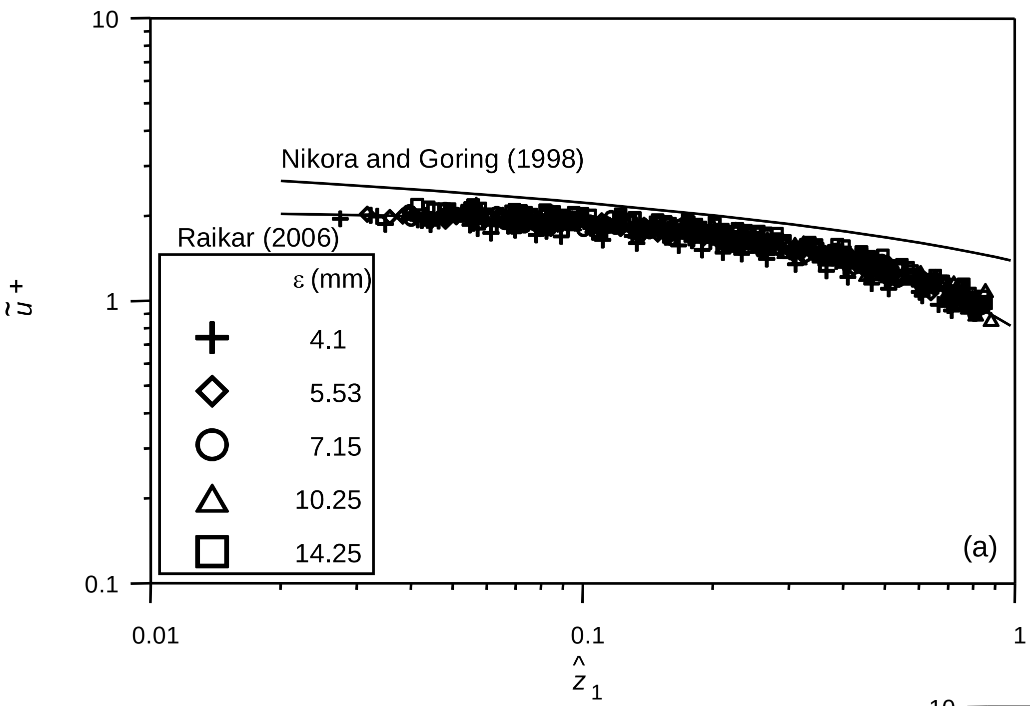
- **Nezu** (1977) suggested the exponential-law for the nondimensional streamwise and vertical turbulence intensities

$$\tilde{u}^+ = B_u \exp(-C_u \hat{z}_1) \qquad (1.35a)$$

$$\tilde{w}^+ = B_w \exp(-C_w \hat{z}_1) \qquad (1.35b)$$

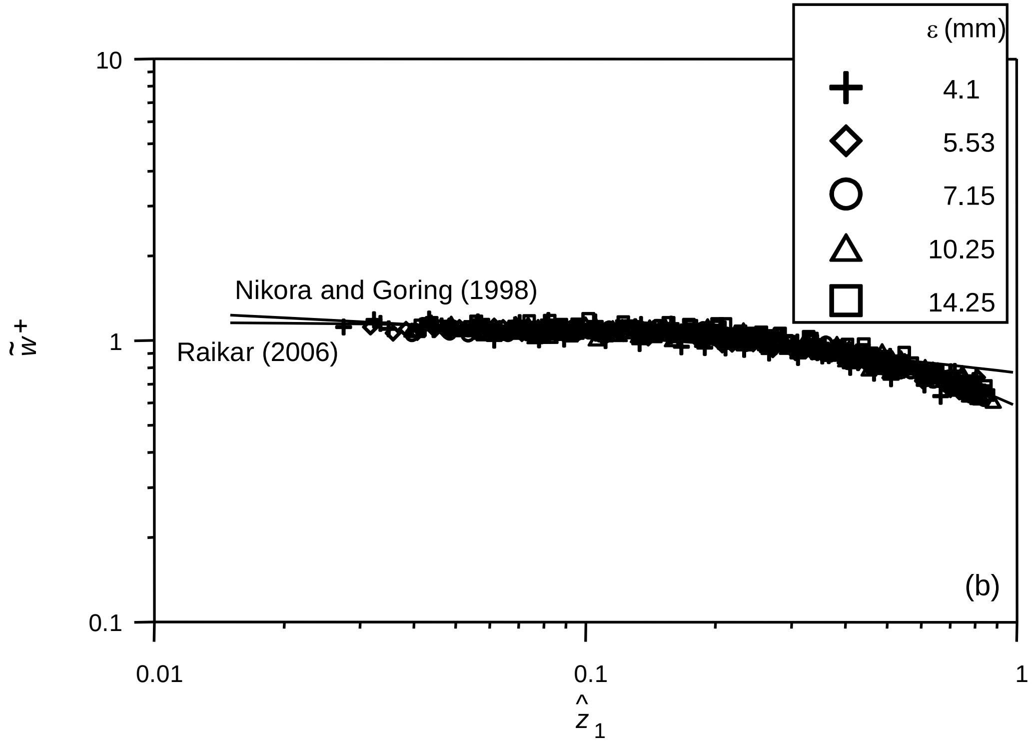
where $\tilde{u}^+ = u^+/u_*$; $\tilde{w}^+ = w^+/w_*$; $w^+ = (\overline{w'w'})^{0.5}$; and B_u, B_w, C_u, and C_w = constants

Source	B_u	C_u	B_w	C_w	Boundary condition
Nezu (1977)	2.3	1	1.27	1	Smooth and rough
Nezu and Rodi (1986)	2.26	0.88	1.23	0.67	Smooth and rough
Cardoso et al. (1989)	2.28	1.08	-	-	Smooth
Kironoto and Graf (1994)	2.04	0.97	1.14	0.76	Rough
Raikar (2006)	2.07	0.95	1.17	0.69	Rough



$\tilde{u}^+$ and $\tilde{w}^+$ for mobile beds (**Nikora and Goring** 1998) are greater than those for beds under near-threshold condition (**Raikar** 2006)

Fig. 1.10 Variations of (a) $\tilde{u}^+$ with $\hat{z}_1$ and (b) $\tilde{w}^+$ with $\hat{z}_1$ for different ε after **Nikora and Goring** (1998) and **Raikar** (2006), and a comparison with the experimental data of **Raikar** (2006) for gravel-beds at near-threshold condition



Experimental Determination of Bed Shear Stress

- The shear velocity u_* [$= (\tau_0/\rho)^{0.5}$] and hence bed shear stress τ_0 can be determined from the well-known *Clauser method*, applying the least square fitting of Eq. (1.32) or Eq. (1.33) to the experimental data in the inner-layer ($\tilde{z} < 0.2$)
- The bed shear stress τ_0 can be obtained from the Reynolds stress profiles extending them on the boundary, that is $\tau_0 = \tau|_{z=z_0}$
- The bed shear stress τ_0 can be calculated from the bed-slope S, as $\tau_0 = \rho g h S$

Bed Shear in a Rectangular Channel with Rough Bed and Smooth Walls

The equation of bed shear stress τ as a function of dynamic pressure

$$\tau_b = \frac{f_b}{8}\rho U_b^2 \qquad (1.37)$$

where f = friction factor

The Colebrook-White equation, used to evaluate f_b

$$\frac{1}{\sqrt{f_b}} = -0.86\ln\left(\frac{\varepsilon P_b}{14.8 A_b} + \frac{2.51}{R_b\sqrt{f_b}}\right) \qquad (1.38)$$

where A = flow area; P = wetted perimeter; and R = Reynolds number of flow

- In a rectangular channel or an experimental flume, the bed is rough consisting of sediment particles and the sidewalls are smooth
- f_w is considerably different from f_b, τ_w is significantly different from τ_b
- **Vanoni**'s (1975) method of *side-wall correction* is applied

Using the continuity equation

$$Q = AU = A_w U_w + A_b U_b \tag{1.39}$$

The equation of force along streamwise direction

$$-A\frac{dp}{dx} = \rho\frac{f}{8}U^2 P = \rho\frac{f_w}{8}U_w^2 P_w + \rho\frac{f_b}{8}U_b^2 P_b \tag{1.40}$$

where *dp*/*dx* = streamwise pressure gradient

Using $U = U_w = U_b$ into Eq. (1.40)

$$Pf = P_w f_w + P_b f_b \tag{1.41}$$

Hydraulic grade line is same for smooth wall and the rough bed regions

$$\frac{Pf}{A} = \frac{P_w f_w}{A_w} = \frac{P_b f_b}{A_b} \tag{1.42}$$

Reynolds numbers of flow for different regions

$$R = \frac{4UA}{\nu\, P}, \quad R_w = \frac{4UA_w}{\nu\, P_w}, \quad R_b = \frac{4UA_b}{\nu\, P_b} \tag{1.43}$$

Using Eq. (1.43) into Eq. (1.42)

$$\frac{R}{f}=\frac{R_w}{f_w}=\frac{R_b}{f_b} \qquad (1.44)$$

As the wall is smooth, the Blasius equation can be used to evaluate f_w

$$f_w=\frac{0.316}{R_w^{0.25}} \qquad (1.45)$$

Using Eqs. (1.39) - (1.45), the following equation is obtained

$$f_b=0.316 R_b\left(\frac{4UA}{\nu\, P_w}-\frac{R_b P_b}{P_w}\right)^{-1.25} \qquad (1.46)$$

Using Eq. (1.43) in Eq. (1.38), Colebrook-White equation becomes

$$\frac{1}{\sqrt{f_b}}=-0.86\ln\left(\frac{\varepsilon U}{3.7\,\nu\, R_b}+\frac{2.51}{R_b\sqrt{f_b}}\right) \qquad (1.47)$$

ε can be assumed as d_{50}, as was done by **Dey** (2003). Unknowns R_b and f_b can be determined numerically solving Eqs. (1.46) and (1.47). Eq. (1.37) is used to estimate the bed shear stress τ_b

Stresses in Nonuniform Unsteady Flow: Dey and Lambert's Approach

- **Dey and Lambert** (2005) developed theory for stresses in nonuniform unsteady flow

The Reynolds equation for two-dimensional non-uniform unsteady flow in open channels

$$\bar{u}\bar{u}_x + \bar{w}\bar{u}_z + \bar{u}_t = \frac{1}{\rho}\left(-p_x + \tau_z\right) \qquad (1.48)$$

where $\bar{u}$ and $\bar{w}$ = time-averaged point velocities in streamwise *x* and normal *z* directions, respectively; *x* and *z* = distances in streamwise and normal directions, respectively; *t* = time; and τ = Reynolds stress at any depth *z*, that is $-\rho\,\overline{u'w'}$

Time-averaged point velocity components and the Reynolds stress

$$\bar{u} = U\psi(\eta, t) \qquad (1.49a)$$

$$\bar{w} = U\varphi(\eta, t) \qquad (1.49b)$$

$$\tau = -\rho\,\overline{u'w'} = \tau\big|_{z=a}\,\xi(\eta, t) \qquad (1.50)$$

where U = depth-averaged velocity; $\tau|_{z=a}$ = bed shear stress; a = zero-velocity level, that is $z|_{\overline{u}=0}$, being equal to 0.033ε; ε = equivalent roughness assumed as d (**Dey** 2003); $\eta = z/h$; and h = flow depth

Prandtl-von Karman universal (logarithmic) velocity distribution law

$$\overline{u} = \frac{1}{\kappa}\sqrt{\frac{\tau|_{z=a}}{\rho}}\ln\left(\frac{z}{a}\right) \tag{1.51}$$

where κ = von Karman constant being 0.4

The depth-averaged velocity U can be given

$$U = \frac{1}{h-a}\int_a^h \overline{u}\,dz = \frac{\beta}{\kappa}\sqrt{\frac{\tau|_{z=a}}{\rho}} \tag{1.52}$$

where $\beta = -[\ln e^{1/(1-e)} + 1]$; and $e = a/h$

Differentiating Eqs. (1.49a) and (1.50)

$$\overline{u}_x = \psi U_x - \frac{U}{h}\eta\psi_\eta h_x \tag{1.53}$$

$$\bar{u}_z = \frac{U}{h}\psi_\eta \qquad (1.54)$$

$$\tau_z = \frac{\tau|_{z=a}}{h}\xi_\eta \qquad (1.55)$$

$$\bar{u}_t = \psi U_t - \frac{U}{h}\eta\psi_\eta h_t + U\psi_t \qquad (1.56)$$

Differentiating Eq. (1.49b)

$$\bar{w}_z = \frac{U}{h}\varphi_\eta \qquad (1.57)$$

Using continuity equation of time-averaged point velocity components, that is $\bar{u}_x$ + $\bar{w}_z$ = 0, and Eq. (1.53)

$$\varphi_\eta = \eta\psi_\eta h_x - \frac{h}{U}\psi U_x \qquad (1.58)$$

Integrating Eq. (1.58)

$$\varphi = h_x \int_e^\eta \eta \psi_\eta d\eta - \frac{h}{U} U_x \int_e^\eta \psi \, d\eta = \psi \eta h_x - \frac{1}{U}(hU_x + Uh_x) \int_e^\eta \psi \, d\eta \tag{1.59}$$

The continuity equation for depth-averaged non-uniform unsteady flow in open channels

$$hU_x + Uh_x + h_t = 0 \tag{1.60}$$

Using Eq. (1.60) into Eq. (1.59), the expression of φ becomes

$$\varphi = \psi \eta h_x + \frac{1}{U} h_t \int_e^\eta \psi \, d\eta \tag{1.61}$$

Inserting Eq. (1.61) in Eq. (1.49b)

$$\overline{w} = \overline{u} \eta h_x + h_t \int_e^\eta \psi \, d\eta \tag{1.62}$$

Substituting Eqs. (1.49a), (1.49b), (1.53) - (1.56) and (1.62) in Eq. (1.48)

$$U\psi^2 U_x + \psi U_t - \frac{U}{h}\left(\eta - \int_e^\eta \psi \, d\eta\right)\psi_\eta h_t + U\psi_t = \frac{1}{\rho}\left(-p_x + \frac{\tau|_{z=a}}{h}\xi_\eta\right) \qquad (1.63)$$

The piezometric pressure gradient is given by

$$p_x = -\rho g(S - h_x) \qquad (1.64)$$

The Saint Venant equation of motion for non-uniform unsteady flow in open channels

$$\frac{U}{g}U_x + h_x - S + \frac{\tau|_{z=a}}{\rho g h} + \frac{1}{g}U_t = 0 \qquad (1.65)$$

- For simplicity, the momentum correction factor is assumed to be unity in Eq. (1.65), as it varies from 1.01 to 1.1 in straight open channels

Rearranging Eq. (1.65)

$$\frac{\rho h U}{\tau|_{z=a}}U_x = -\frac{\rho g h}{\tau|_{z=a}}\left(h_x - S + \frac{1}{g}U_t\right) - 1 = -\lambda - 1 \qquad (1.66)$$

where λ = streamwise pressure gradient parameter

$$\lambda = \frac{\rho g h}{\tau|_{z=a}}\left(h_x - S + \frac{1}{g}U_t\right) \tag{1.67}$$

- In Eq. (1.67), for steady flow U_t = 0; and for uniform flow h_x = 0. In accelerating and decelerating flows λ < -1 and λ > -1

Using Eqs. (1.66) and (1.67) into Eq. (1.63) yields

$$-(\lambda+1)\psi^2 + (\psi-1)\frac{\rho h}{\tau|_{z=a}}U_t - \left(\eta - \int_e^\eta \psi\, d\eta\right)\frac{\rho U}{\tau|_{z=a}}\psi_\eta h_t + \frac{\rho h}{\tau|_{z=a}}U\psi_t = -\lambda + \xi_\eta \tag{1.68}$$

Dividing Eq. (1.51) by Eq. (1.52) and equating to Eq. (1.49a)

$$\frac{\bar{u}}{U} = \frac{1}{\beta}\ln\left(\frac{\eta}{e}\right) = \psi \tag{1.69}$$

- Eq. (1.69) represents the velocity profile characteristics that remain independent of time (ψ_t = 0)

Substituting Eq. (1.69) into Eq. (1.68) and making $\psi_t = 0$

$$\xi_\eta = \lambda - (\lambda+1)\frac{1}{\beta^2}\ln^2\left(\frac{\eta}{e}\right) + \left[\frac{1}{\beta}\ln\left(\frac{\eta}{e}\right) - 1\right]\frac{\rho h}{\tau|_{z=a}}U_t - \frac{1}{\beta}\left\{1 - \frac{1}{\beta}\left[\ln\left(\frac{\eta}{e}\right) + \frac{e}{\eta} - 1\right]\right\}\frac{\rho U}{\tau|_{z=a}}h_t \tag{1.70}$$

At the bed ($\eta = e$)

$$\xi_\eta\Big|_{\eta=e} = \lambda - \frac{\rho h}{\tau|_{z=a}}U_t - \frac{1}{\beta}\cdot\frac{\rho U}{\tau|_{z=a}}h_t \tag{1.71}$$

Integrating Eq. (1.71) and using the boundary condition $\xi|_{\eta=\varepsilon} = 1$

$$\xi = 1 + \lambda(\eta - e) - (\lambda+1)\frac{1}{\beta^2}\left\{\eta\ln^2\left(\frac{\eta}{e}\right) - 2\left[\eta\ln\left(\frac{\eta}{e}\right) - \eta + e\right]\right\}$$
$$+ \left[\frac{\eta}{\beta}\ln\left(\frac{\eta}{e}\right) - \left(1 + \frac{1}{\beta}\right)(\eta - e)\right]\frac{\rho h}{\tau|_{z=a}}U_t + \frac{1}{\beta}\left[\frac{\eta + e}{\beta}\ln\left(\frac{\eta}{e}\right) - \left(1 + \frac{2}{\beta}\right)(\eta - e)\right]\frac{\rho U}{\tau|_{z=a}}h_t \tag{1.72}$$

Substituting λ from Eq. (1.67) and U_x from Eq. (1.60) into Eq. (1.72), the equation of non-dimensional Reynolds stress for non-uniform unsteady flow in open channels

$$\xi = 1 + (\eta - e)\frac{\rho g h}{\tau|_{z=a}}(h_x - S) - \frac{1}{\beta^2}\left\{\eta \ln^2\left(\frac{\eta}{e}\right) - 2\left[\eta \ln\left(\frac{\eta}{e}\right) - \eta + e\right]\right\}\frac{\rho U}{\tau|_{z=a}}(U h_x + h_t)$$

$$+\frac{1}{\beta}\left[\eta \ln\left(\frac{\eta}{e}\right) - \eta + e\right]\frac{\rho h}{\tau|_{z=a}}U_t + \frac{1}{\beta}\left[\frac{\eta + e}{\beta}\ln\left(\frac{\eta}{e}\right) - \left(1 + \frac{2}{\beta}\right)(\eta - e)\right]\frac{\rho U}{\tau|_{z=a}}h_t \qquad (1.73)$$

The bed shear stress $\tau|_{z=a}$ can be obtained from Eq. (1.73) using the boundary condition $\tau|_{z=h} = 0$

$$\tau|_{z=a} = -(1-e)\rho g h(h_x - S) + \frac{1}{\beta^2}[\ln^2 e + 2(\ln e + 1 - e)]\rho U(U h_x + h_t) + \frac{1}{\beta}(\ln e + 1 - e)\rho h U_t$$

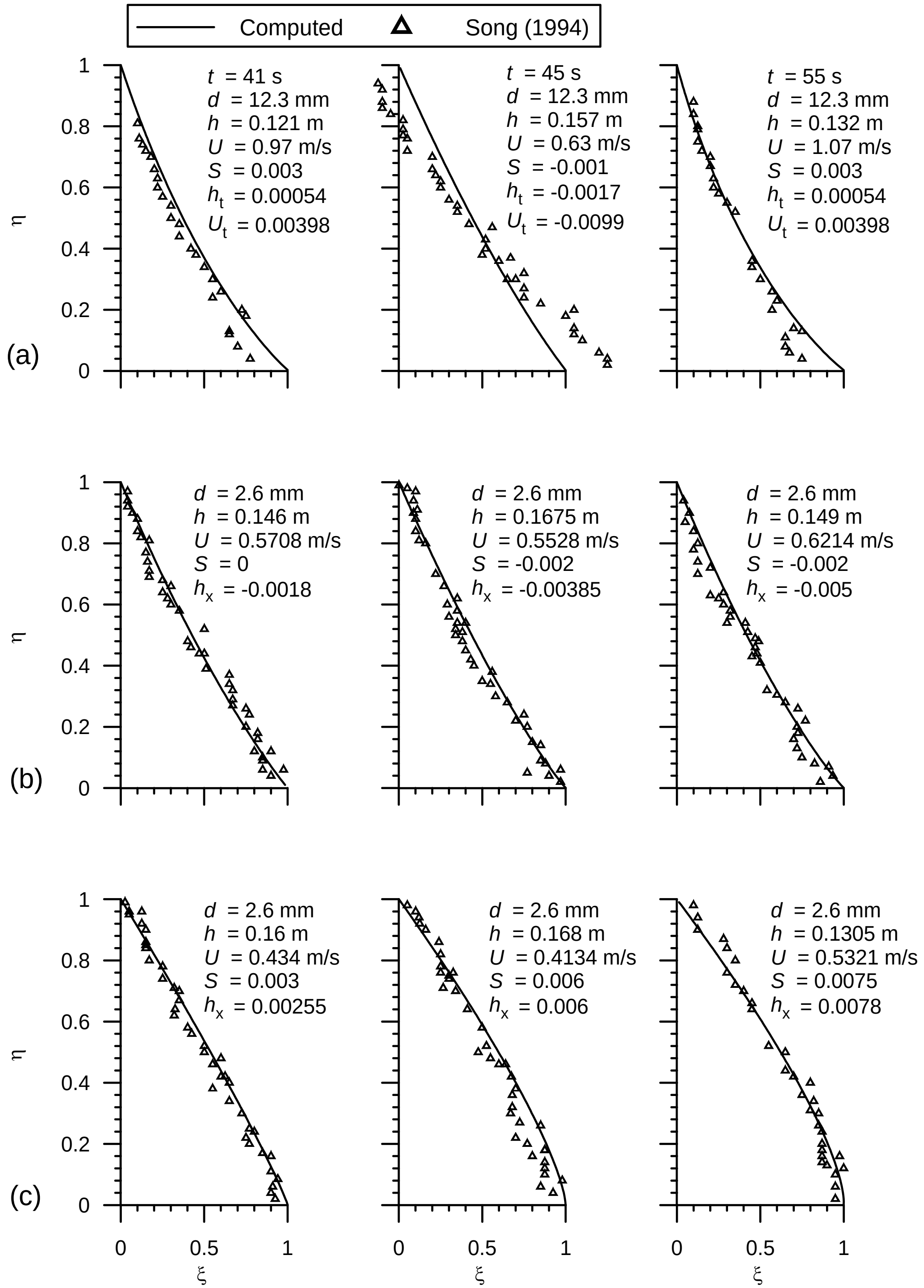
$$+\frac{1}{\beta}\left[\frac{1+e}{\beta}\ln\varepsilon + \left(1 + \frac{2}{\beta}\right)(1-e)\right]\rho U h_t \qquad (1.74)$$

Using Eqs. (1.73) and (1.74), the equation of Reynolds stress $\tau|_{z=z}$ can be obtained

$$\tau|_{z=z} = -(1-\eta)\rho g h(h_x - S) - \frac{1}{\beta^2}\left\{\eta \ln^2\left(\frac{\eta}{e}\right) - \ln^2 e - 2\left[\eta \ln\left(\frac{\eta}{e}\right) + \ln e + 1 - \eta\right]\right\}\rho U(U h_x + h_t)$$

$$+\frac{1}{\beta}\left[\eta \ln\left(\frac{\eta}{e}\right) + \ln e + 1 - \eta\right]\rho h U_t + \frac{1}{\beta}\left\{\frac{1}{\beta}\left[(\eta + e)\ln\left(\frac{\eta}{e}\right) + (1+e)\ln e\right] + \left(1 + \frac{2}{\beta}\right)(1-\eta)\right\}\rho U h_t$$

(1.75)



- The distribution of Reynolds stress in non-dimensional and dimensional form can be computed using Eqs. (1.73) and (1.75)
- However, Eq. (1.74) can be used to estimate bed shear stress $\tau|_{z=a}$
- The computed curves are in good agreement with the experimental data of **Song** (1994) for unsteady and non-uniform flows

Fig. 1.11 Non-dimensional Reynolds stress profiles and comparisons with the data of **Song** (1994): (a) unsteady; (b) non-uniform accelerating; and (c) non-uniform decelerating flows

Turbulent Burst

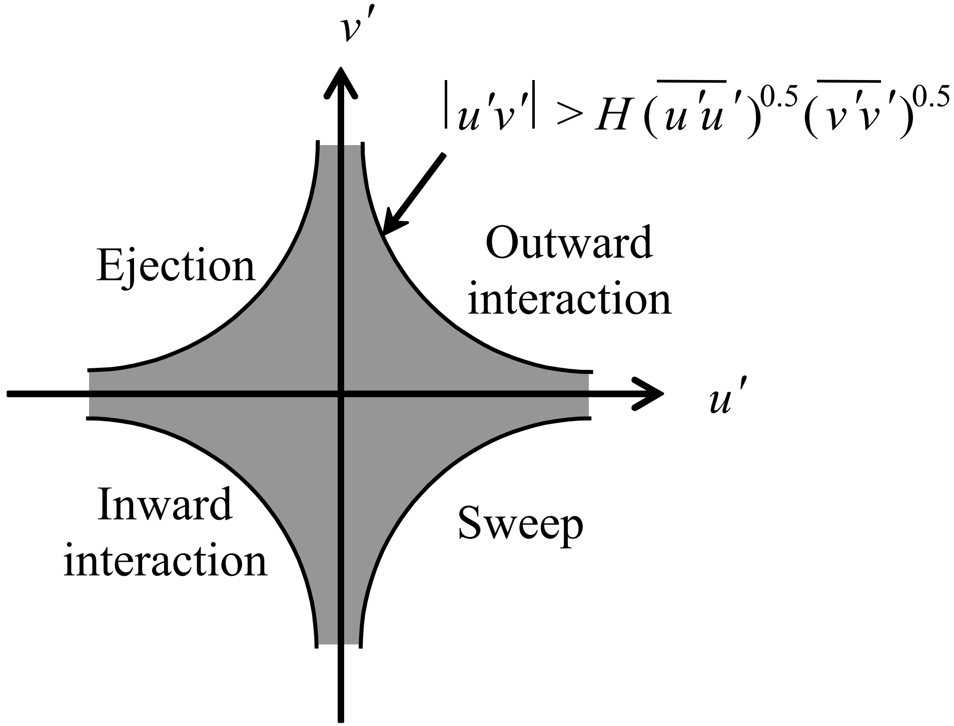
A flow structure dominated by viscosity consists of large 3D high- and low-speed velocity steaks

The near-bed region of flow has an extremely complex structure and most of the turbulence is produced there

Turbulent bursts is formed by the emission of fluid from the low-speed streaks

The sequence is described by ejection and sweep which have an important role on entrainment of bed sediments. During the ejection, the upward flow expands the shear layer and the associated small-scale flow structures to a broad region. It occurs as a low-speed fluid streak that oscillates in three dimensions lifts up from the bed and then collapse to entrain into the main body of flow. The ejected fluid which remains as a result of retardation is brushed away by high-speed fluid that approaches the bed in a process called the sweep. During sweep, the downward flow generates a narrow, highly turbulent shear layer containing multiple small-scale vortices.

Quadrant Analysis



The hyperbolic shaded zone bounded by the curve $|u'v'|$ = constant is called a *hole*. Introducing a parameter *H* called *hole-size* that represents threshold level

The conditional stochastic analysis can be performed introducing a detection function $\lambda_{i,H}(t)$ defined as

$$\lambda_{i,H}(y,t) = \begin{cases} 1, & \text{if } (u', v') \text{ is in quadrant } i \text{ and if } |u'v'| > H(\overline{u'u'})^{0.5}(\overline{v'v'})^{0.5} \\ 0, & \text{otherwise} \end{cases}$$

At any point, contributions to the total Reynolds shear stress from the quadrant *i* outside the hyperbolic hole region of size *H* is given by

$$\langle u'v' \rangle_{i,H} = \lim_{T \to 0} \frac{1}{T} \int_0^T u'(t) v'(t) \lambda_{i,H}(y,t) dt$$

Thank You